

Name (print legibly): **Solutions**

8 March 2007

Math 142

### Quiz 6

*Directions:* You have 20 minutes to complete the quiz. Please show all relevant steps - if you show no work, I cannot give you partial credit. No calculators are allowed during the quiz. This quiz is worth 15 points.

1. (5 Points) Complete the following table and then draw the triangles associated with each substitution.

| If We See   | We Try to Substitute |
|-------------|----------------------|
| $x^2 - a^2$ | $x = a \sec \theta$  |
| $a^2 - x^2$ | $x = a \sin \theta$  |
| $x^2 + a^2$ | $x = a \tan \theta$  |

Please see your notes for the triangles.

2. (10 Points) Compute  $\int \frac{dx}{x^2\sqrt{9x^2-4}}$ .

*Solution.* We first note that  $9x^2 - 4 = 9(x^2 - \frac{4}{9})$  so that

$$\int \frac{dx}{x^2\sqrt{9x^2-4}} = \frac{1}{3} \int \frac{dx}{x^2\sqrt{x^2 - \frac{4}{9}}}$$

Now, let  $a = \frac{2}{3}$  and  $x = \frac{2}{3} \sec \theta$ . Then  $dx = \frac{2}{3} \sec \theta \tan \theta d\theta$ . Also, for appropriate<sup>1</sup> values of  $\theta$ , we get  $\sqrt{x^2 - \frac{4}{9}} = \sqrt{\frac{4}{9} \tan^2 \theta} = \frac{2}{3} \tan \theta$ . We substitute this back into the integral to get

$$\frac{1}{3} \int \frac{\frac{2}{3} \sec \theta \tan \theta}{\frac{4}{9} \sec^2 \theta \frac{2}{3} \tan \theta} d\theta = \frac{1}{3} \cdot \frac{9}{4} \int \frac{d\theta}{\sec \theta} = \frac{3}{4} \int \cos \theta d\theta = \frac{3}{4} \sin \theta + C$$

We draw a triangle to see that

$$\sin \theta = \frac{\sqrt{x^2 - \frac{4}{9}}}{x} = \frac{\sqrt{9x^2 - 4}}{3x}$$

Then

$$\int \frac{dx}{x^2\sqrt{9x^2-4}} = \frac{3}{4} \cdot \frac{\sqrt{9x^2-4}}{3x} + C = \frac{\sqrt{9x^2-4}}{4x} + C$$

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<sup>1</sup>It is necessary to choose  $\theta$  so that  $\tan \theta$  is positive so we are not left with an absolute value.