

Math 142  
Spring 2007

### Quiz 3 Solutions

This week's quiz was the following: find the length of  $f(x) = \sqrt{1-x^2}$  on  $[-\frac{1}{2}, \frac{1}{2}]$ . First, we compute the derivative and see that

$$f'(x) = \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}}$$

so that  $[f'(x)]^2 = \frac{x^2}{1-x^2}$  and

$$1 + [f'(x)]^2 = 1 + \frac{x^2}{1-x^2} = \frac{1}{1-x^2}$$

Thus

$$L = \int_{-1/2}^{1/2} \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) \Big|_{-1/2}^{1/2} = \frac{\pi}{3}$$