Name (write legibly): **Solutions**
1 February 2007
Math 142

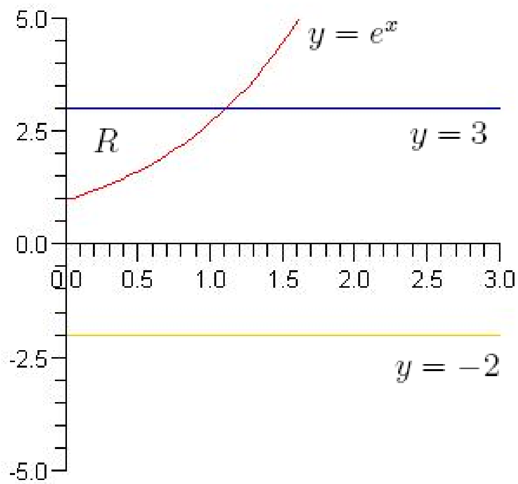
Quiz 2

Directions: You have 20 minutes to complete the quiz. Please show all relevant steps - if you show no work, I cannot give you partial credit. No calculators are allowed during the quiz. This quiz is worth 15 points.

For problems 1 and 2, let R be the region in the first quadrant bounded between $y = 3$ and $y = e^x$.

1. (7.5 Points) Using disks/washers, set up, but DO NOT evaluate an integral which gives the volume of the solid generated if we revolve the region R about: (a) the y-axis and (b) the line $y = -2$.

Solution. For both parts, we clearly have $e^x = 3$ when $x = \ln 3$. As usual, we need to plot the situation at hand. We have:



For part (a), the outter radius of a typical disk is $R(y) = \ln y$. Thus

$$V = \pi \int_1^3 (\ln y)^2 dy$$

For part (b), the outter radius is given by $R(x) = 3 + 2 = 5$ and the inner radius is $r(x) = e^x + 2$. Then

$$V = \pi \int_0^{\ln 3} [5^2 - (e^x + 2)^2] dx = \pi \int_0^{\ln 3} (25 - e^{2x} - 4e^x - 4) dx = \pi \int_0^{\ln 3} (21 - e^{2x} - 4e^x) dx$$

2. (7.5 Points) Using cylindrical shells, repeat problem 1. Again, you only need to set up the integral.

Solution. For part (a), we revolve about the y-axis so the shell radius is $r(x) = x$. The height of the shell is $h(x) = 3 - e^x$ and x is varying from $x = 0$ to $x = \ln 3$. Thus

$$V = 2\pi \int_0^{\ln 3} x(3 - e^x)dx$$

For part (b), we revolve about $y = -2$ so the shell radius is $r(y) = y + 2$. Also, the height of each shell is $h(y) = \ln y$ and y varies from $y = 1$ to $y = 3$ so that

$$V = 2\pi \int_1^3 (y + 2) \ln y \; dy$$