Name (write legibly): **Solutions**
25 January 2007
Math 142

Quiz 1

Directions: You have 20 minutes to complete the quiz. Please show all relevant steps - if you show no work, I cannot give you partial credit. No calculators are allowed during the quiz. This quiz is worth 15 points.

1. (10 Points) Compute the following:

(a) $\displaystyle\int_0^{\sqrt{\pi/2}} x\cos(x^2)dx$

(b) $\displaystyle\int \frac{7x^3}{1+x^4}dx$

We evaluate both of these by substitution. For (a), let $u = x^2$ so that $du = 2x\ dx$. When $x = 0$, $u = 0$ and when $x = \sqrt{\pi/2}$, $u = \pi/2$. Thus

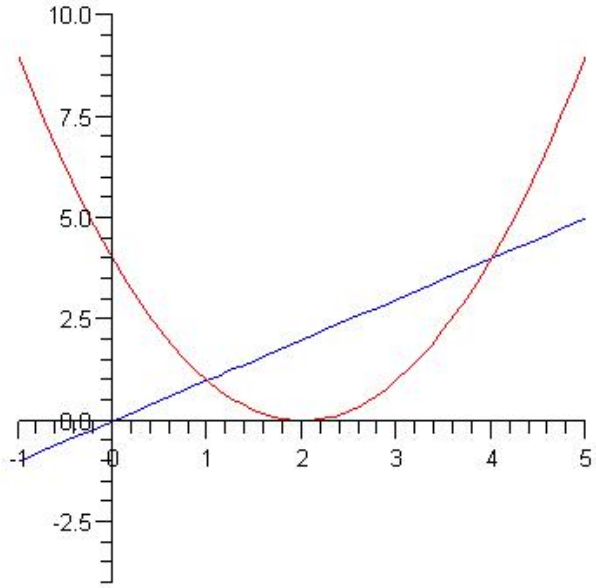
$$\int_0^{\sqrt{\pi/2}} x\cos(x^2)dx = \frac{1}{2}\int_0^{\pi/2} \cos u\ du = \frac{1}{2}\sin u\ \Big|_0^{\pi/2} = \frac{1}{2}$$

For (b), let $u = 1 + x^4$ so that $du = 4x^3\ dx$ and

$$\int \frac{7x^3}{1+x^4}dx = \frac{7}{4}\int \frac{du}{u} = \frac{7}{4}\ln u + C = \frac{7}{4}\ln(1+x^4) + C$$

2. (5 Points) Set up, but do not evaluate, an integral which gives the area of the region bounded between $f(x) = (x-2)^2$ and $g(x) = x$.

The first thing we do is plot the situation at hand:



Firstly, we solve $(x-2)^2 = x$ so $x^2 - 4x + 4 = x$ or $x^2 - 5x + 4 = (x-1)(x-4) = 0$ so that $x = 1$ and $x = 4$. Then the area is given by

$$\begin{aligned} A &= \int_1^4 [x - (x-2)^2]dx = \int_1^4 [x - (x^2 - 4x + 4)]dx = \int_1^4 (-x^2 + 5x - 4)dx \\ &= -\frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x\Big|_1^4 = \left(-\frac{64}{3} + 40 - 16\right) - \left(-\frac{1}{3} + \frac{5}{2} - 4\right) = \frac{8}{3} + \frac{11}{6} \\ &= \frac{9}{2} \end{aligned}$$

It is not necessary for you to evaluate the integral above. I am mainly grading to see that your integral is correct. I included the answer for the sake of being complete.