Irreducibility of Classical Polynomials and their Generalizations

Irreducibility of Classical Polynomials and their Generalizations

by Michael Filaseta

Irreducibility of Classical Polynomials and their Generalizations

by Michael Filaseta

University of South Carolina

Irreducibility:

A polynomial $f(x) \in \mathbb{Q}[x]$ is *irreducible* provided

- $f(x)$ degree at least 1,
- $f(x)$ does not factor as a product of two polynomials in $\mathbb{Q}[x]$ each of degree ≥ 1.

Some Goals of the Talk:

Some Goals of the Talk:

- Give a general discussion of the irreducibility of some classical polynomials

Some Goals of the Talk:

- Give a general discussion of the irreducibility of some classical polynomials
- Show connections to

Some Goals of the Talk:

- Give a general discussion of the irreducibility of some classical polynomials
- Show connections to
  - problems in the distribution of primes

Some Goals of the Talk:

- Give a general discussion of the irreducibility of some classical polynomials
- Show connections to
  - problems in the distribution of primes
  - diophantine questions

Some Goals of the Talk:

- Give a general discussion of the irreducibility of some classical polynomials
- Show connections to
  - problems in the distribution of primes
  - diophantine questions
  - transcendence type results

Some Goals of the Talk:

- Give a general discussion of the irreducibility of some classical polynomials
- Show connections to
  - problems in the distribution of primes
  - diophantine questions
  - transcendence type results
  - Galois theory

Some Goals of the Talk:

- Give a general discussion of the irreducibility of some classical polynomials
- Show connections to
  - problems in the distribution of primes
  - diophantine questions
  - transcendence type results
  - Galois theory
  - wavelets

Some Polynomials to be Discussed:

- Hermite Polynomials
- Laguerre Polynomials
- Bessel Polynomials

Some Polynomials NOT to be Discussed:

- Chebyshev Polynomials (too easy)
- Bernoulli Polynomials (except for a special case)
- Legendre Polynomials

Email from Mark Kon:

Given a function $f(x)$, its wavelet transform consists of the family of functions $g(2^j x) * f(x)$, where g is the gaussian function, and j is an integer. The question was: if we know the zeroes of the second derivatives of this family of functions (over all j), can we recover f? ... The problem reduces to showing that none of these polynomials [certain Hermite polynomials] has zeroes (aside from the trivial one at the origin) which coincides with a zero of another one. So the bottom line is that the conjecture that f is uniquely recoverable follows from the non-overlapping of the zeroes of the Hermite polynomials.

Email from Mark Kon:

Given a function $f(x)$, its wavelet transform consists of the family of functions $g(2^j x) * f(x)$, where g is the gaussian function, and j is an integer. The question was: if we know the zeroes of the second derivatives of this family of functions (over all j), can we recover f? ... The problem reduces to showing that none of these polynomials [certain Hermite polynomials] has zeroes (aside from the trivial one at the origin) which coincides with a zero of another one. So the bottom line is that the conjecture that f is uniquely recoverable follows from the non-overlapping of the zeroes of the Hermite polynomials.