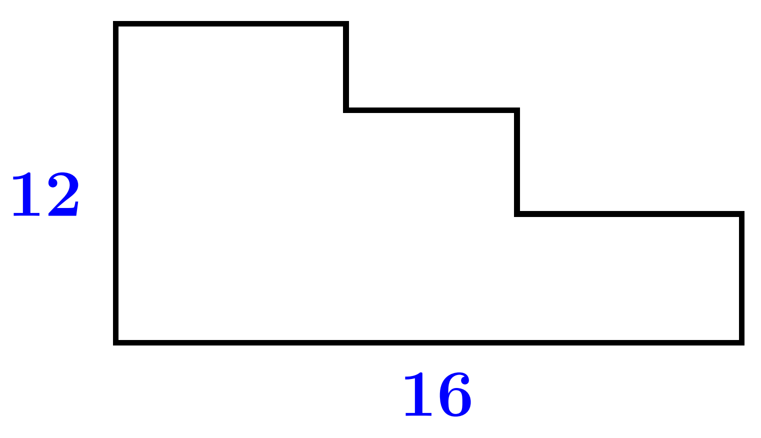
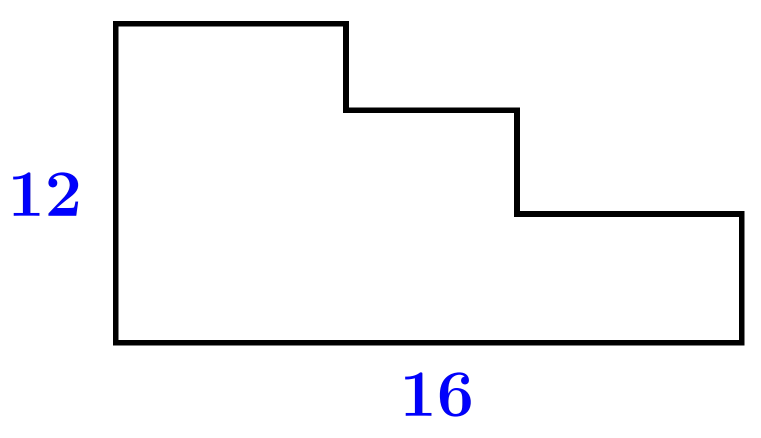
Problem 4:

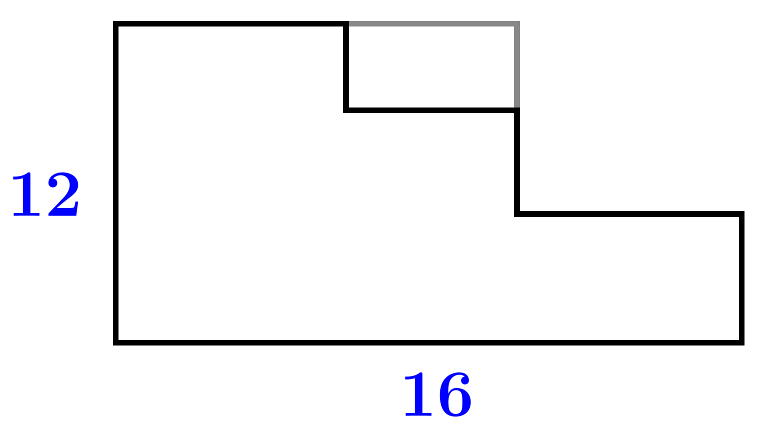


Problem 4:



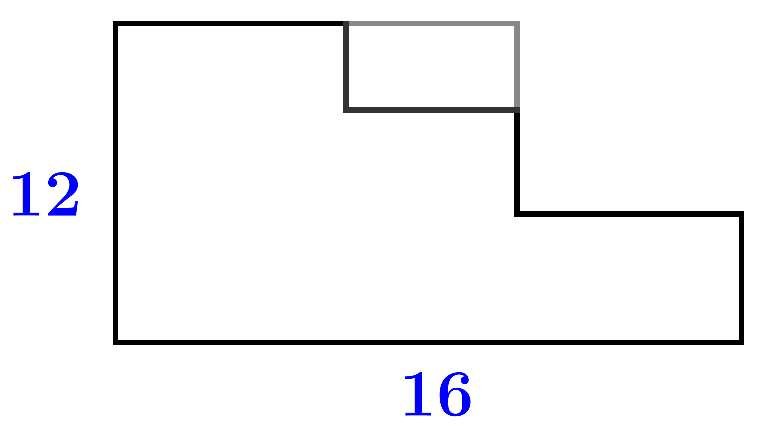
What’s the perimeter?

Problem 4:



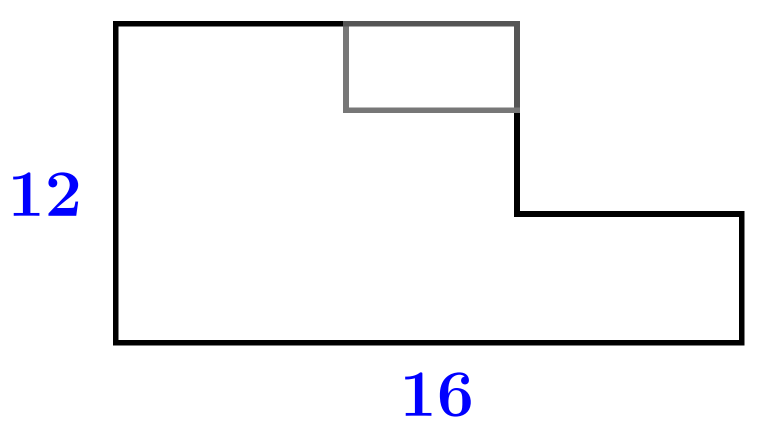
What’s the perimeter?

Problem 4:



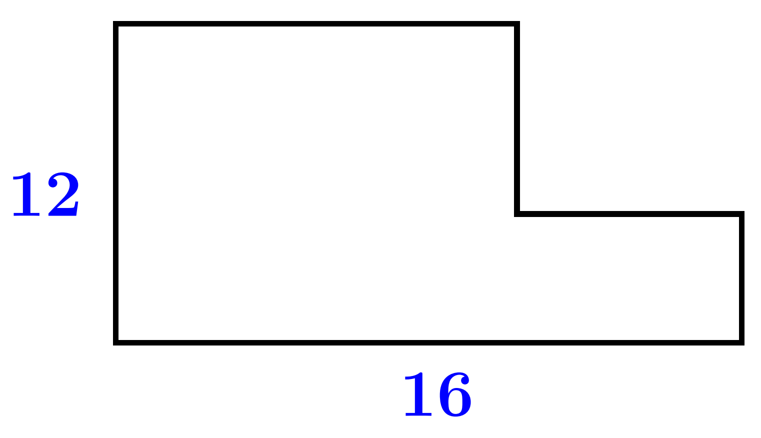
What’s the perimeter?

Problem 4:



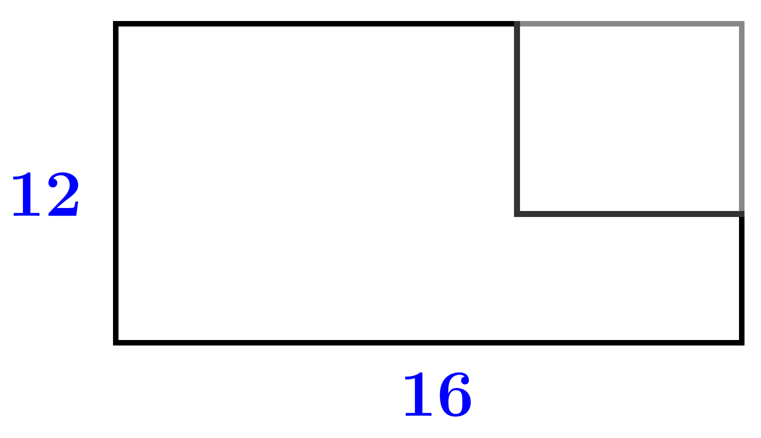
What’s the perimeter?

Problem 4:



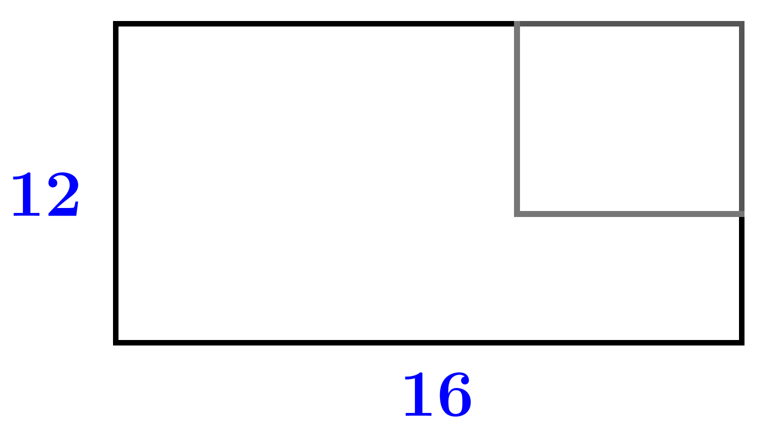
What's the perimeter?

Problem 4:



What’s the perimeter?

Problem 4:



What’s the perimeter?

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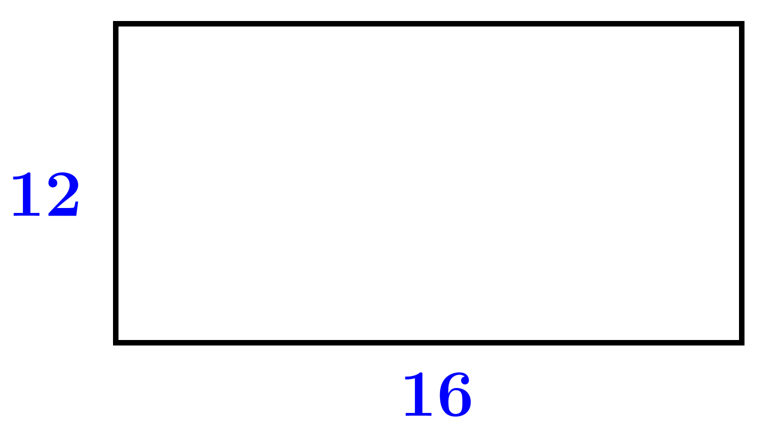

What’s the perimeter?

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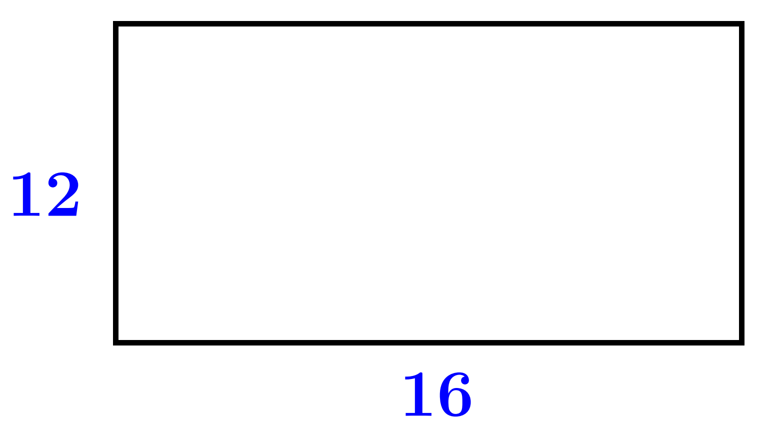
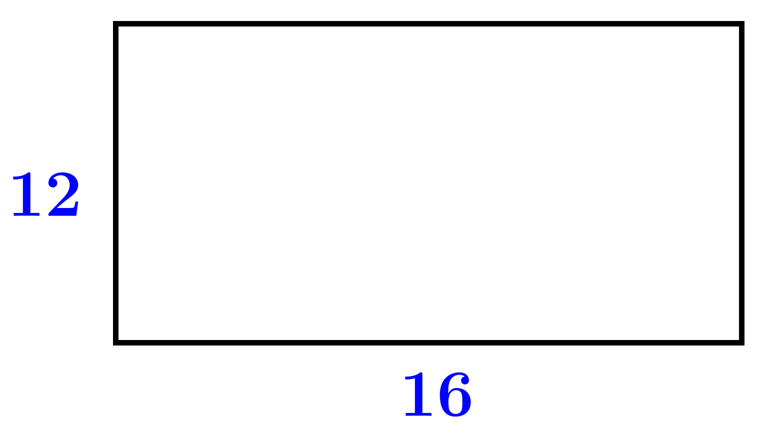


What's the perimeter?

$$2 \times (12 + 16) = 56$$

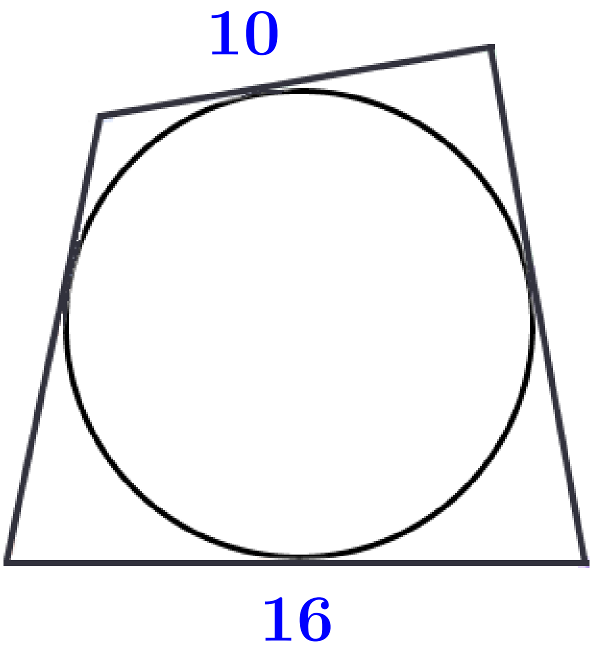
Problem 4:



What’s the perimeter?

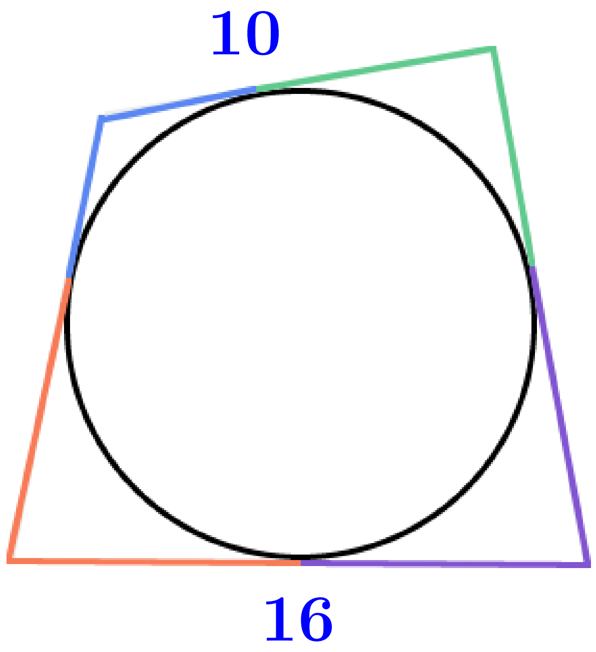
$$2 \times (12 + 16) = 56$$

Problem 9:



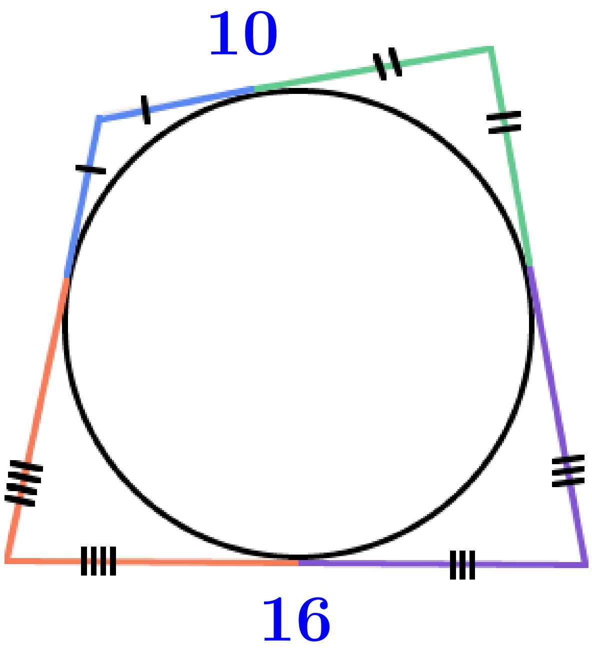
What’s the perimeter of this quadrilateral?

Problem 9:



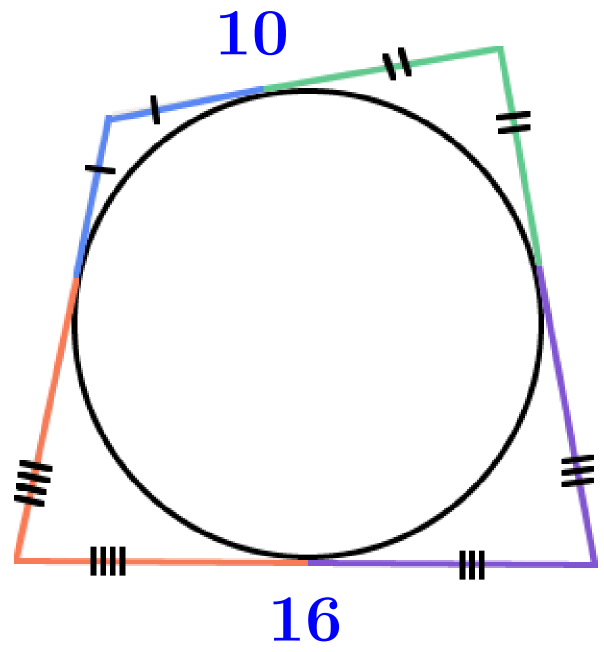
What’s the perimeter of this quadrilateral?

Problem 9:



What’s the perimeter of this quadrilateral?

Problem 9:



What’s the perimeter of this quadrilateral? 52

Problem 12:

$$a^2 - b^2 = 2003$$

Problem 12:

$$a^2 - b^2 = 2003 \implies a^2 + b^2 = ?$$

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$$a > 0 \qquad b > 0$$

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$$a^2 - b^2 = 2003 \implies a^2 + b^2 = ?$$

$$a > b \qquad b > 0$$

$$2003 = a^2 - b^2$$

Problem 12:

$$a^2 - b^2 = 2003 \implies a^2 + b^2 = ?$$

$$a > b \qquad b > 0$$

$$2003 = a^2 - b^2 = (a - b)(a + b)$$

Problem 12:

$$a^2 - b^2 = 2003 \implies a^2 + b^2 = ?$$

$$a > b \qquad b > 0$$

$$\underbrace{2003}_{\substack{\uparrow \\ \text{prime}}} = a^2 - b^2 = (a-b)(a+b)$$

Problem 12:

$$a^2 - b^2 = 2003 \implies a^2 + b^2 = ?$$

$$a > b \qquad b > 0$$

$$\underbrace{2003}_{\substack{\uparrow \\ \text{prime}}} = a^2 - b^2 = (\underbrace{a-b}_{\uparrow})(\underbrace{a+b}_{\uparrow})$$

positive integers

Problem 12:

$$a^2 - b^2 = 2003 \implies a^2 + b^2 = ?$$

$$a > b \qquad b > 0$$

$$2003 = a^2 - b^2 = (a-b)(a+b)$$

$$a - b = 1$$
$$a + b = 2003$$

Problem 12:

$$a^2 - b^2 = 2003 \implies a^2 + b^2 = ?$$

$$a > b \qquad b > 0$$

$$2003 = a^2 - b^2 = (a-b)(a+b)$$

$$\left.\begin{array}{l} a - b = 1 \\ a + b = 2003 \end{array}\right\} \implies \begin{array}{r} a = 1002 \\ b = 1001 \end{array}$$

Problem 12:

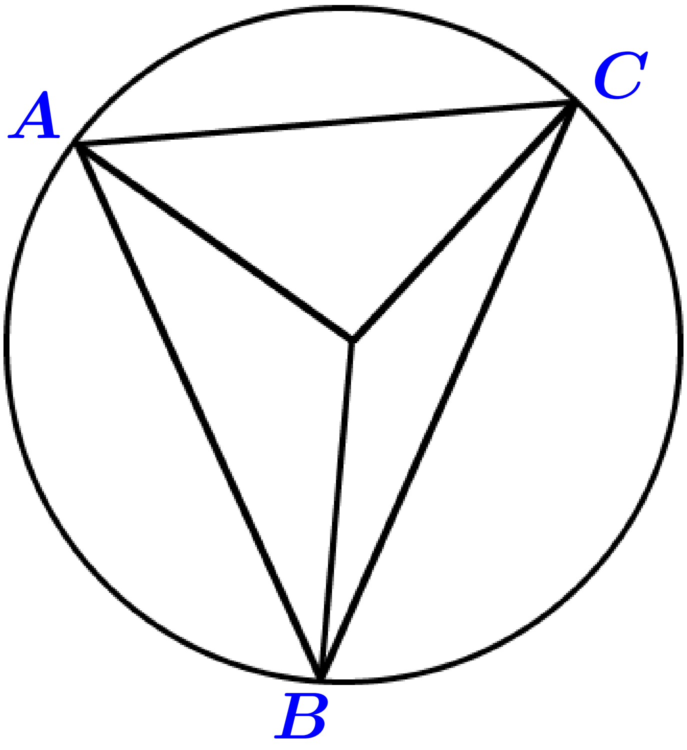
$$a^2 - b^2 = 2003 \implies a^2 + b^2 = 2006005$$

$$a > b \qquad b > 0$$

$$2003 = a^2 - b^2 = (a-b)(a+b)$$

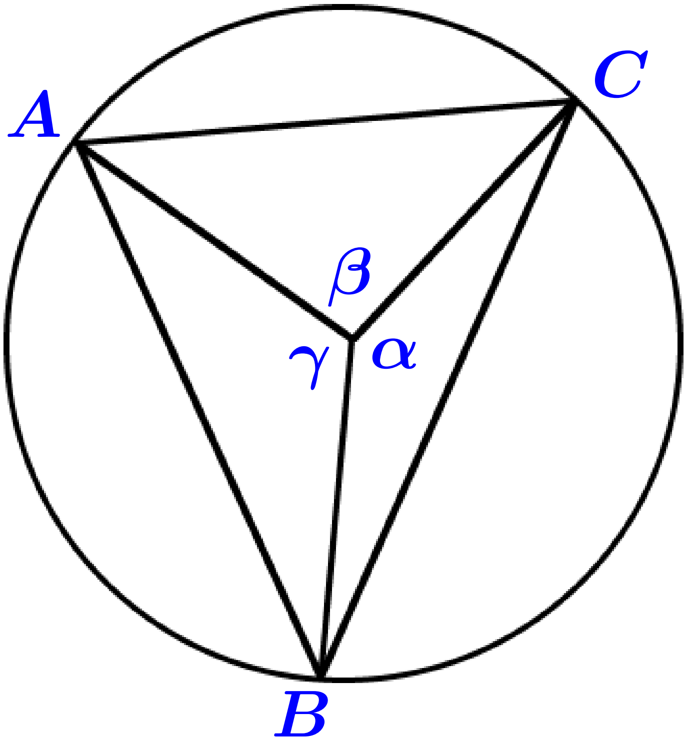
$$\left.\begin{aligned} a - b &= 1 \\ a + b &= 2003 \end{aligned}\right\} \implies \begin{aligned} a &= 1002 \\ b &= 1001 \end{aligned}$$

Problem 18:



area of circle is $\mathbf{20}$ area of $\triangle ABC$ is $\mathbf{8}$

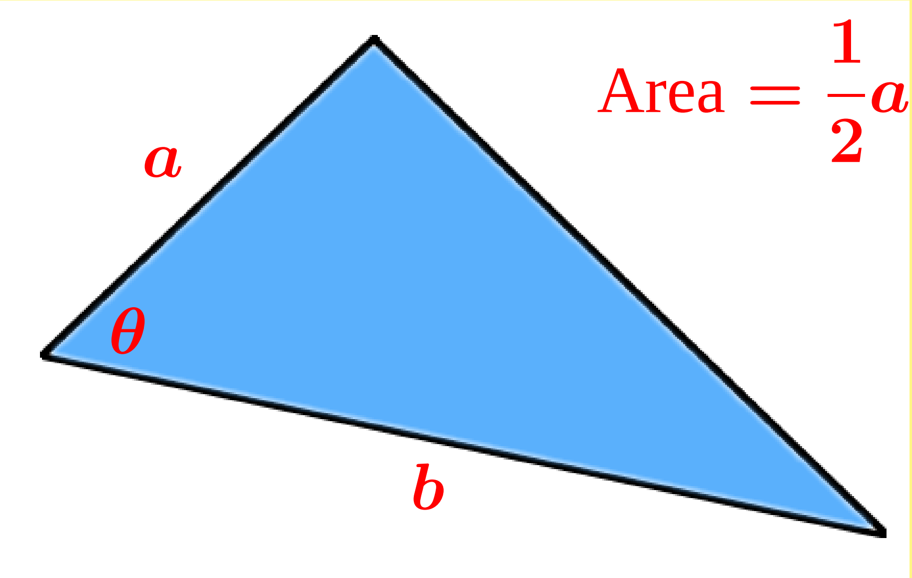
Problem 18:



area of circle is $\mathbf{20}$ area of $\triangle ABC$ is $\mathbf{8}$

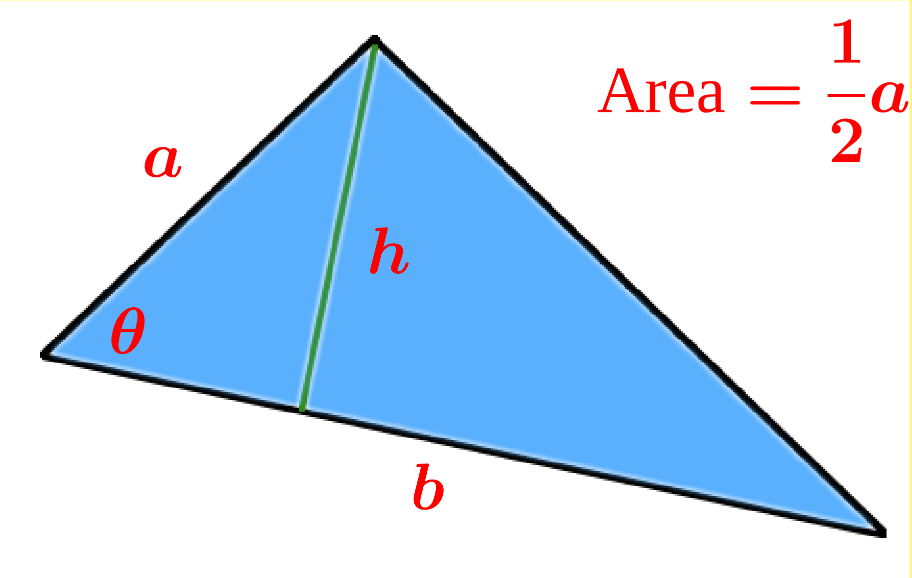
Calculate $\sin\alpha + \sin\beta + \sin\gamma$.

Problem 18:



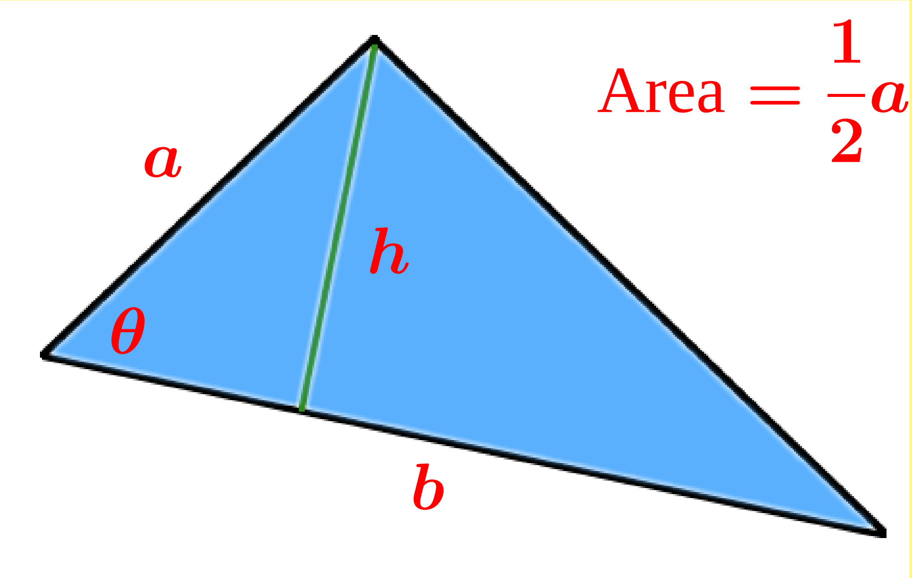
$$\text{Area} = \frac{1}{2}ab\sin\theta$$

Problem 18:



$$\text{Area} = \frac{1}{2} ab \sin \theta$$

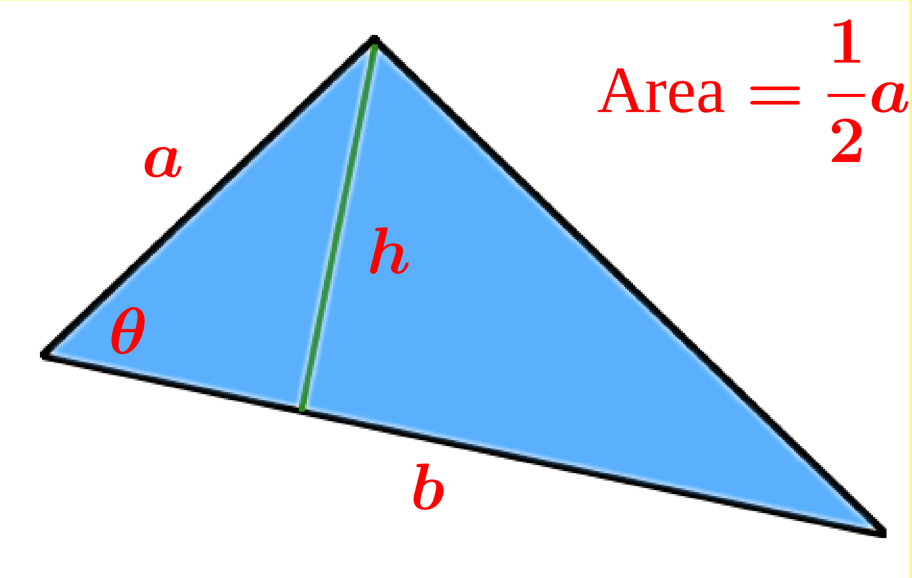
Problem 18:



$$\text{Area} = \frac{1}{2}ab\sin\theta$$

$$\text{Area} = \frac{1}{2}hb$$

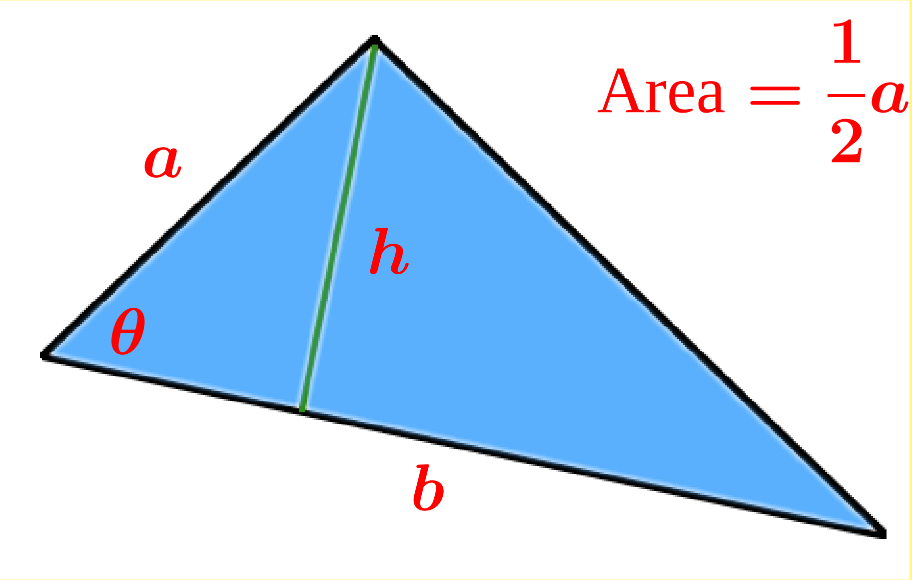
Problem 18:



$$\text{Area} = \frac{1}{2}ab\sin\theta$$

$$\text{Area} = \frac{1}{2}hb = \frac{1}{2}(a\sin\theta)b$$

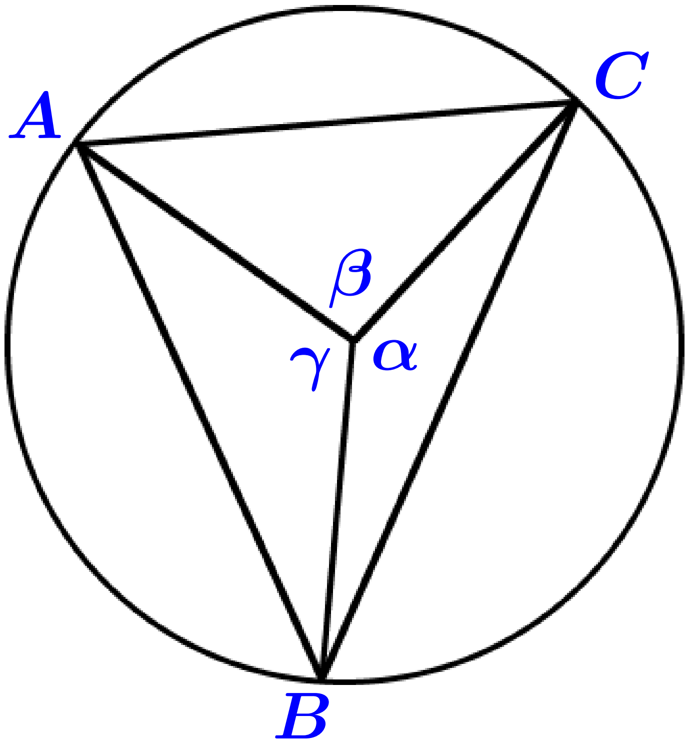
Problem 18:



$$\text{Area} = \frac{1}{2}ab\sin\theta$$

$$\text{Area} = \frac{1}{2}hb = \frac{1}{2}(a\sin\theta)b = \frac{1}{2}ab\sin\theta$$

Problem 18:



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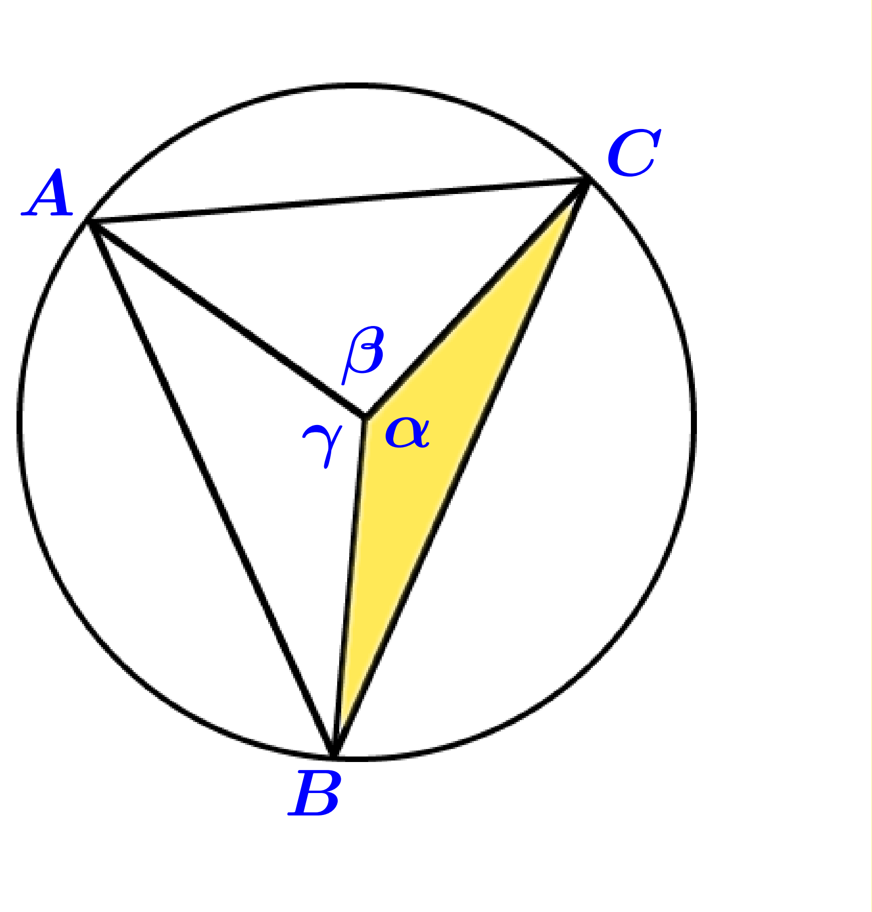

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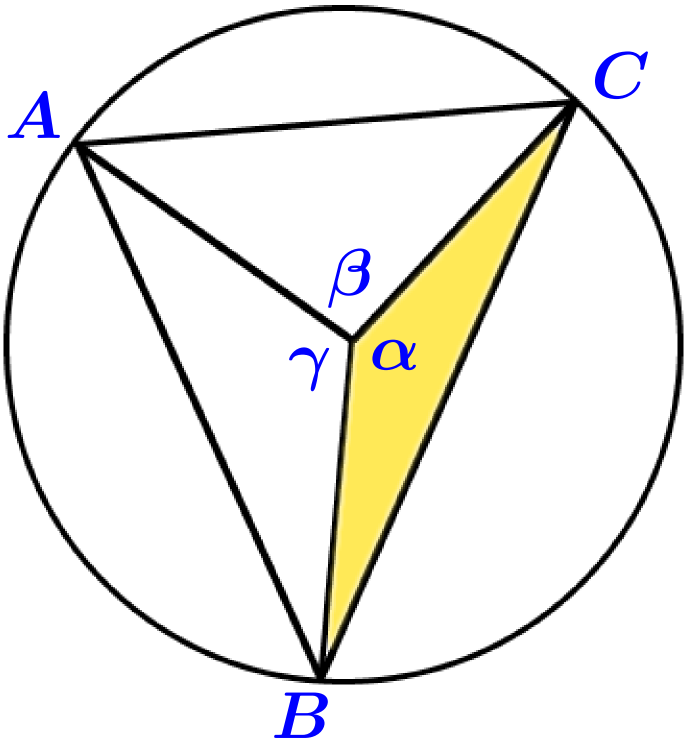
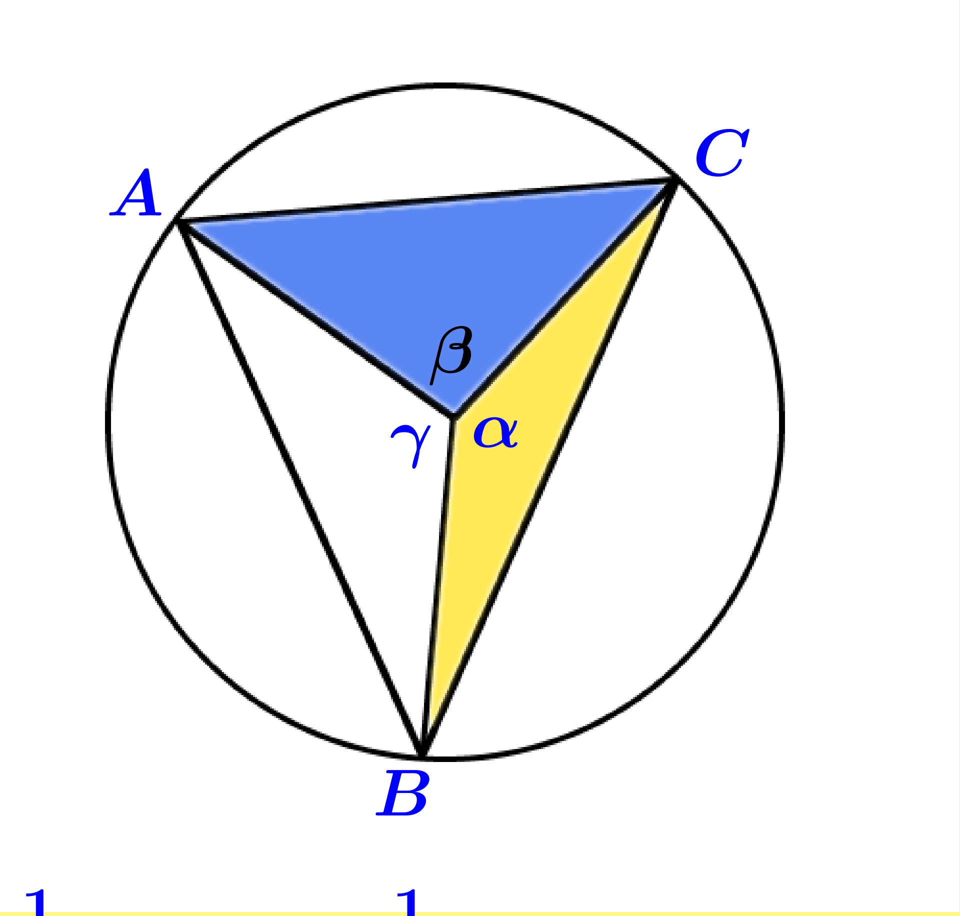


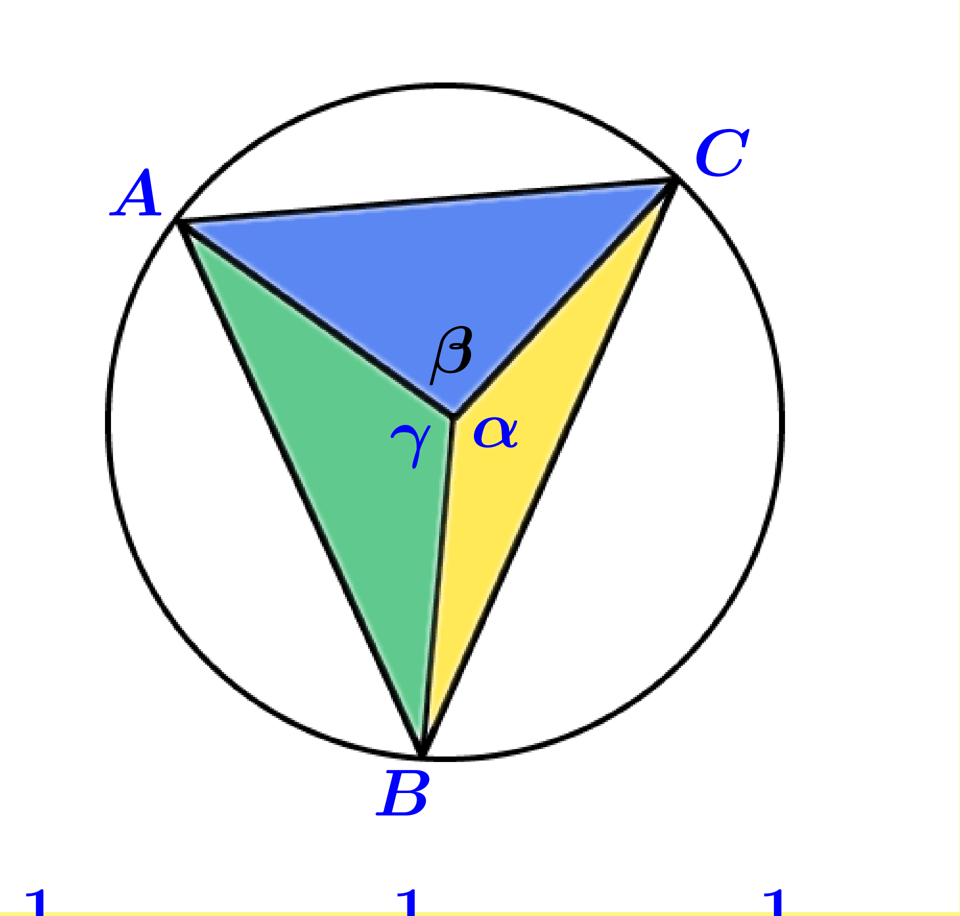
$$\text{Area} = \frac{1}{2} r^2 \sin\alpha$$

Problem 18:



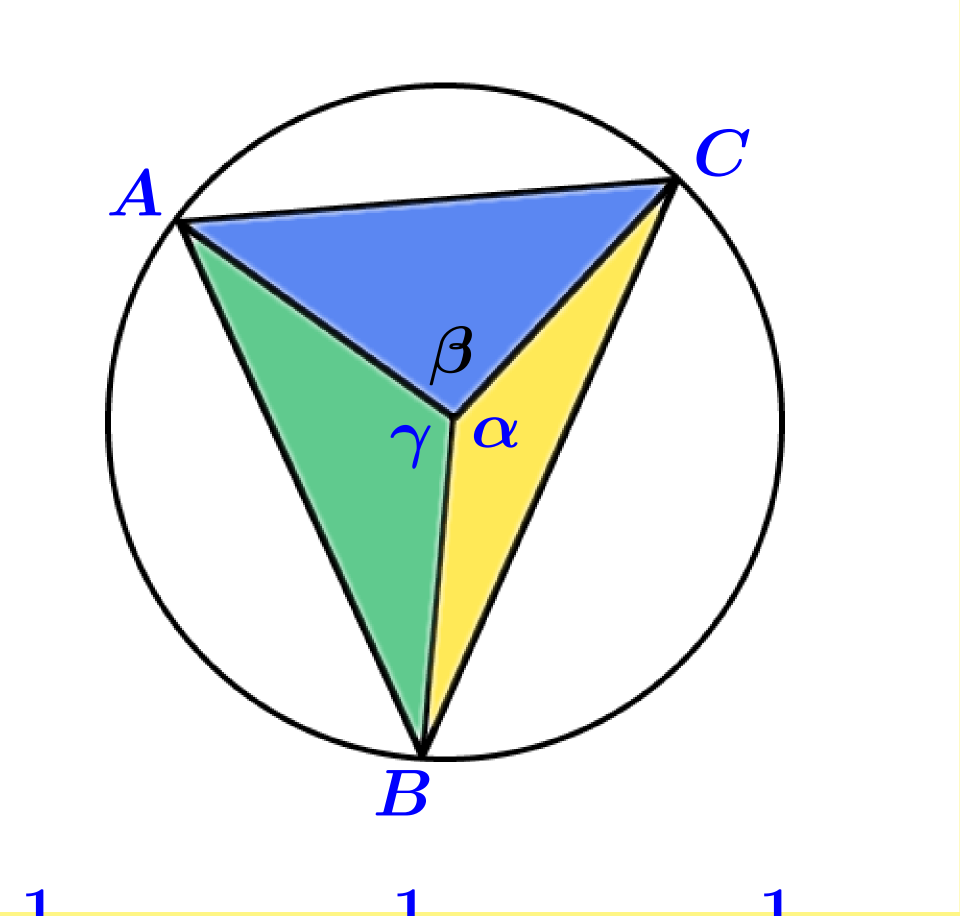
$$\text{Area} = \frac{1}{2}r^2 \sin\alpha + \frac{1}{2}r^2 \sin\beta$$

Problem 18:



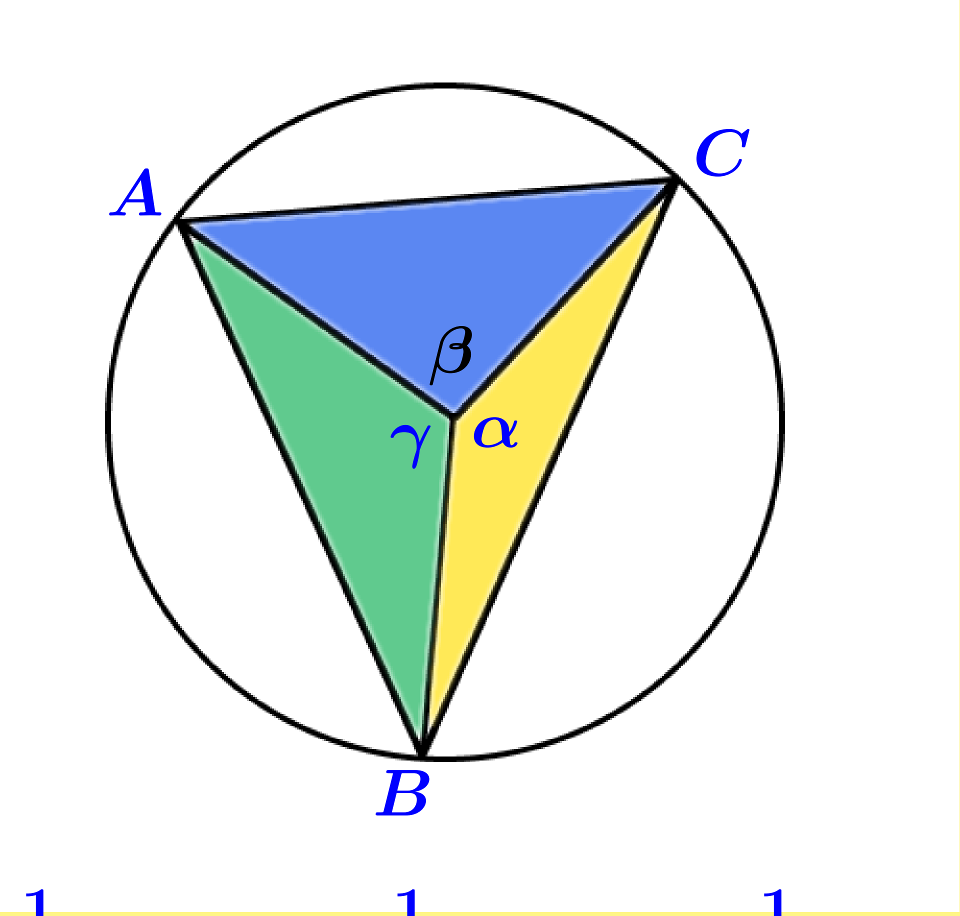
$$\text{Area} = \frac{1}{2}r^2\sin\alpha + \frac{1}{2}r^2\sin\beta + \frac{1}{2}r^2\sin\gamma$$

Problem 18:



$$8 = \frac{1}{2}r^2 \sin\alpha + \frac{1}{2}r^2 \sin\beta + \frac{1}{2}r^2 \sin\gamma$$

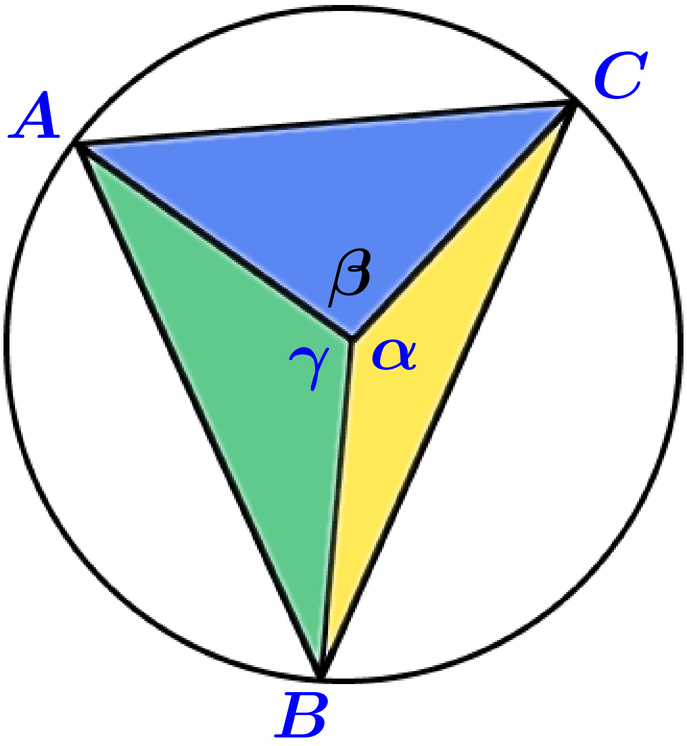
Problem 18:



$$8 = \frac{1}{2}r^2 \sin\alpha + \frac{1}{2}r^2 \sin\beta + \frac{1}{2}r^2 \sin\gamma$$

$$\sin\alpha + \sin\beta + \sin\gamma = \frac{16}{r^2}$$

Problem 18:



$$\pi r^2 = 20$$

$$8 = \frac{1}{2}r^2 \sin\alpha + \frac{1}{2}r^2 \sin\beta + \frac{1}{2}r^2 \sin\gamma$$

$$\sin\alpha + \sin\beta + \sin\gamma = \frac{16}{r^2}$$

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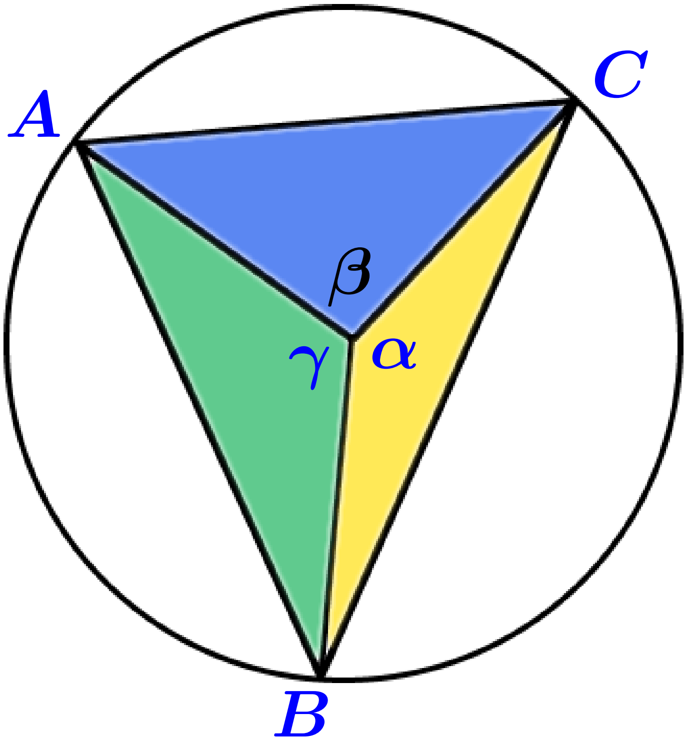

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$$\sin\alpha + \sin\beta + \sin\gamma = \frac{16}{r^2} = \frac{16\pi}{20}$$

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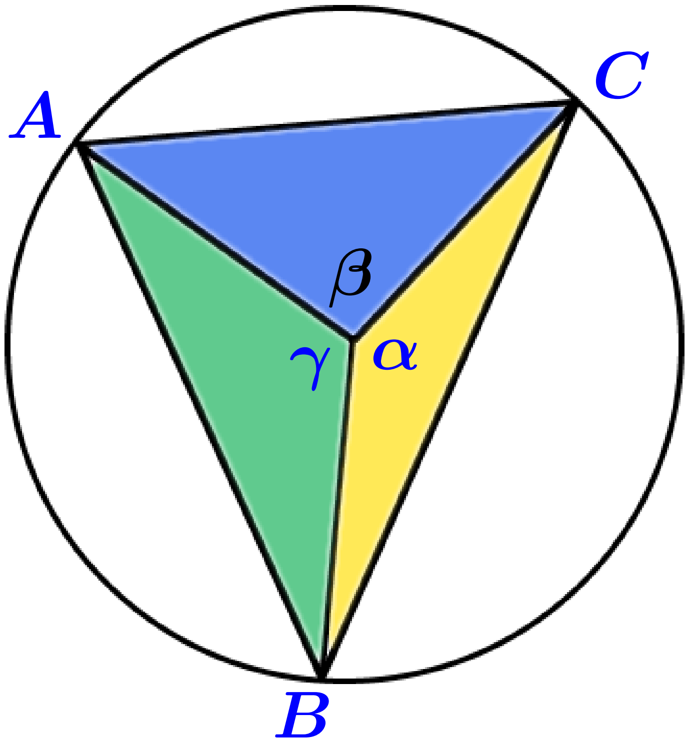

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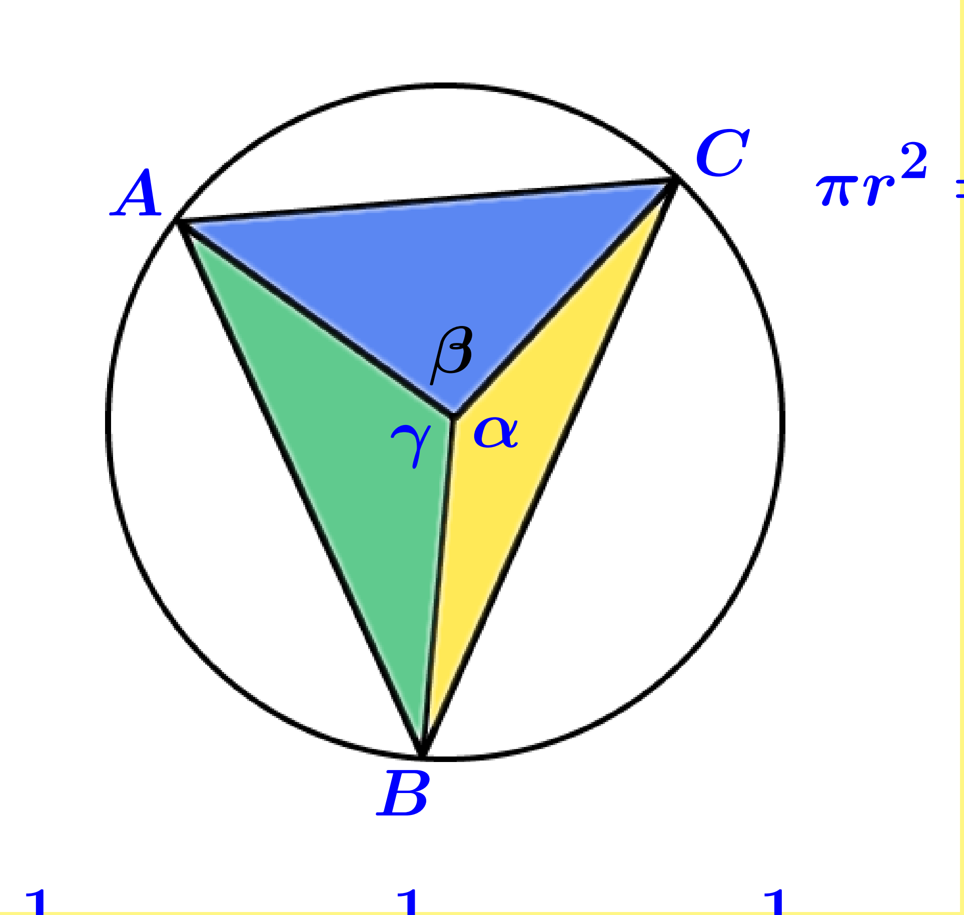

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$$\sin\alpha + \sin\beta + \sin\gamma = \frac{16}{r^2} = \frac{16\pi}{20} = \frac{4\pi}{5}$$

Problem 19:

$$
\begin{aligned}
a + b^2 + 2ac &= 29 \\
b + c^2 + 2ab &= 18 \\
c + a^2 + 2bc &= 25
\end{aligned}
$$

Problem 19:

$$a + b^2 + 2ac = 29$$
$$b + c^2 + 2ab = 18$$
$$c + a^2 + 2bc = 25$$

What's $a + b + c$?

Problem 19:

$$\begin{aligned} a + b^2 + 2ac &= 29 \\ b + c^2 + 2ab &= 18 \\ c + a^2 + 2bc &= 25 \end{aligned}$$

$a + b + c$

Problem 19:

$$\begin{aligned} a + b^2 + 2ac &= 29 \\ b + c^2 + 2ab &= 18 \\ c + a^2 + 2bc &= 25 \end{aligned}$$

$$a + b + c + (a + b + c)^2$$

Problem 19:

$$a + b^2 + 2ac = 29$$
$$b + c^2 + 2ab = 18$$
$$c + a^2 + 2bc = 25$$

$$a + b + c + (a + b + c)^2 = 72$$

Problem 19:

$$a+b^2+2ac=29$$
$$b+c^2+2ab=18$$
$$c+a^2+2bc=25$$

$$a+b+c+(a+b+c)^2=72$$

$a+b+c$ is a positive root of $x^2+x-72=0$

Problem 19:

$$\begin{aligned} a+b^2+2ac &= 29 \\ b+c^2+2ab &= 18 \\ c+a^2+2bc &= 25 \end{aligned}$$

$$a+b+c+(a+b+c)^2=72$$

$$a+b+c \text{ is a positive root of } \underbrace{x^2+x-72=0}_{\substack{\uparrow \\ (x-8)(x+9)=0}}$$

Problem 19:

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Problem 19:

$$a + b^2 + 2ac = 29$$
$$b + c^2 + 2ab = 18$$
$$c + a^2 + 2bc = 25$$

$$a = 4 \qquad b = 1 \qquad c = 3$$

Problem 23:

$$(100x + 10y + z)^2 = (x + y + z)^5$$

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x, y, and z are non-zero digits

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$$(100x + 10y + z)^2 = (x + y + z)^5$$

x, y, and z are non-zero digits

$$x^2 + y^2 + z^2 = \ ?$$

Problem 23:

$$N = (100x + 10y + z)^2 = (x + y + z)^5$$

Problem 23:

$$\boldsymbol{N = (100x + 10y + z)^2 = (x + y + z)^5}$$

$\boldsymbol{N}$ is both a square and a fifth power

Problem 23:

$$N = (100x + 10y + z)^2 = (x + y + z)^5$$

N is both a square and a fifth power

N is a tenth power

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$$N = (\overbrace{100x + 10y + z}^{\substack{\text{3 digit number} \\ \downarrow}})^2 = (x + y + z)^5$$

N is both a square and a fifth power

N is a tenth power

Problem 23:

$$N = (100x + 10y + z)^2 = (x + y + z)^5$$

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$$2^{10} = 32^2 \qquad 3^{10} = 243^2 \qquad 4^{10} = 1024^2$$

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$$x = 2,\ y = 4,\ z = 3$$

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N is both a square and a fifth power

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$$2^{10} = 32^2 \qquad 3^{10} = 243^2 \qquad 4^{10} = 1024^2$$

$$x = 2,\ y = 4,\ z = 3 \implies x^2 + y^2 + z^2 = 29$$

Problem 24:

N divided by $\mathbf{5}$ gives a remainder of $\mathbf{2}$
N divided by $\mathbf{7}$ gives a remainder of $\mathbf{3}$
N divided by $\mathbf{9}$ gives a remainder of $\mathbf{4}$

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N divided by $\mathbf{5}$ gives a remainder of $\mathbf{2}$
N divided by $\mathbf{7}$ gives a remainder of $\mathbf{3}$
N divided by $\mathbf{9}$ gives a remainder of $\mathbf{4}$

What is the smallest such N?

Problem 24:

N divided by $\mathbf{5}$ gives a remainder of $\mathbf{2}$

N divided by $\mathbf{7}$ gives a remainder of $\mathbf{3}$

N divided by $\mathbf{9}$ gives a remainder of 4

Problem 24:

$\boldsymbol{N}$ divided by $\mathbf{5}$ gives a remainder of $\mathbf{2}$

$\boldsymbol{N}$ divided by $\mathbf{7}$ gives a remainder of $\mathbf{3}$

$\boldsymbol{N}$ divided by $\mathbf{9}$ gives a remainder of $\mathbf{4}$

$$\boldsymbol{N = 5k + 2}$$

Problem 24:

$\boldsymbol{N}$ divided by $\mathbf{5}$ gives a remainder of $\mathbf{2}$
$\boldsymbol{N}$ divided by $\mathbf{7}$ gives a remainder of $\mathbf{3}$
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$$\boldsymbol{N = 5k + 2 \implies 2N + 1 = 10k + 5}$$

Problem 24:

$\boldsymbol{N}$ divided by $\mathbf{5}$ gives a remainder of $\mathbf{2}$

$\boldsymbol{N}$ divided by $\mathbf{7}$ gives a remainder of $\mathbf{3}$

$\boldsymbol{N}$ divided by $\mathbf{9}$ gives a remainder of 4

$$\begin{aligned} \boldsymbol{N = 5k + 2} &\implies \boldsymbol{2N + 1 = 10k + 5} \\ &\implies \boldsymbol{2N + 1} \text{ is divisible by } \mathbf{5} \end{aligned}$$

Problem 24:

$\boldsymbol{N}$ divided by $\mathbf{5}$ gives a remainder of $\mathbf{2}$
$\boldsymbol{N}$ divided by $\mathbf{7}$ gives a remainder of $\mathbf{3}$
$\boldsymbol{N}$ divided by $\mathbf{9}$ gives a remainder of 4

$$\begin{aligned} \boldsymbol{N = 7k + 3} &\implies \boldsymbol{2N + 1 = 14k + 7} \\ &\implies \boldsymbol{2N + 1} \text{ is divisible by } \mathbf{7} \end{aligned}$$

Problem 24:

$\boldsymbol{N}$ divided by $\mathbf{5}$ gives a remainder of $\mathbf{2}$
$\boldsymbol{N}$ divided by $\mathbf{7}$ gives a remainder of $\mathbf{3}$
$\boldsymbol{N}$ divided by $\mathbf{9}$ gives a remainder of $\mathbf{4}$

$$\boldsymbol{N = 9k + 4 \implies 2N + 1 = 18k + 9}$$

$$\boldsymbol{\implies 2N + 1} \text{ is divisible by } \boldsymbol{9}$$

Problem 24:

N divided by 5 gives a remainder of 2
N divided by 7 gives a remainder of 3
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$2N + 1$ is divisible by 5, 7, and 9

Problem 24:

N divided by 5 gives a remainder of 2
N divided by 7 gives a remainder of 3
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$$2N+1 \text{ is divisible by } \underbrace{5,\ 7, \text{ and } 9}_{\substack{\uparrow \\ \text{no common prime factors}}}$$

Problem 24:

N divided by 5 gives a remainder of 2
N divided by 7 gives a remainder of 3
N divided by 9 gives a remainder of 4

$2N + 1$ is divisible by 5, 7, and 9

$$2N + 1 \geq 5 \cdot 7 \cdot 9 = 315$$

Problem 24:

$\boldsymbol{N}$ divided by $\mathbf{5}$ gives a remainder of $\mathbf{2}$
$\boldsymbol{N}$ divided by $\mathbf{7}$ gives a remainder of $\mathbf{3}$
$\boldsymbol{N}$ divided by $\mathbf{9}$ gives a remainder of 4

$\mathbf{2}\boldsymbol{N} + \mathbf{1}$ is divisible by $\mathbf{5}$, $\mathbf{7}$, and $\mathbf{9}$

$$\mathbf{2}\boldsymbol{N} + \mathbf{1} \geq \mathbf{5} \cdot \mathbf{7} \cdot \mathbf{9} = \mathbf{315} \implies \boldsymbol{N} \geq \mathbf{157}$$

Problem 24:

$\boldsymbol{N}$ divided by $\mathbf{5}$ gives a remainder of $\mathbf{2}$
$\boldsymbol{N}$ divided by $\mathbf{7}$ gives a remainder of $\mathbf{3}$
$\boldsymbol{N}$ divided by $\mathbf{9}$ gives a remainder of $\mathbf{4}$

$\mathbf{2}\boldsymbol{N} + \mathbf{1}$ is divisible by $\mathbf{5}$, $\mathbf{7}$, and $\mathbf{9}$

$$\mathbf{2}\boldsymbol{N} + \mathbf{1} \geq \mathbf{5} \cdot \mathbf{7} \cdot \mathbf{9} = \mathbf{315} \implies \boldsymbol{N} \geq \mathbf{157}$$

$\boldsymbol{N} = \mathbf{157}$ works

Problem 24:

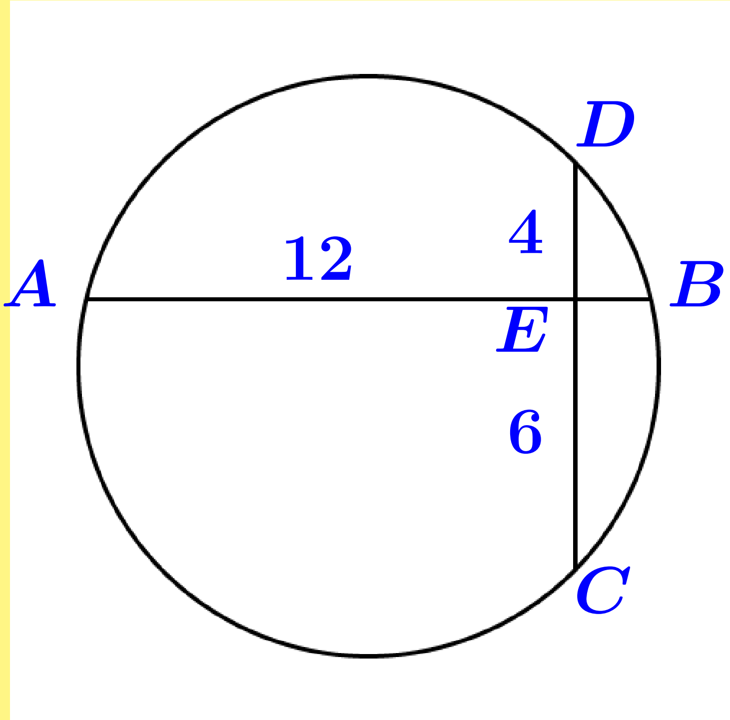
$\boldsymbol{N}$ divided by $\mathbf{5}$ gives a remainder of $\mathbf{2}$
$\boldsymbol{N}$ divided by $\mathbf{7}$ gives a remainder of $\mathbf{3}$
$\boldsymbol{N}$ divided by $\mathbf{9}$ gives a remainder of 4

$\mathbf{2}\boldsymbol{N} + \mathbf{1}$ is divisible by $\mathbf{5}$, $\mathbf{7}$, and $\mathbf{9}$

$$\mathbf{2}\boldsymbol{N} + \mathbf{1} \geq \mathbf{5} \cdot \mathbf{7} \cdot \mathbf{9} = \mathbf{315} \implies \boldsymbol{N} \geq \mathbf{157}$$

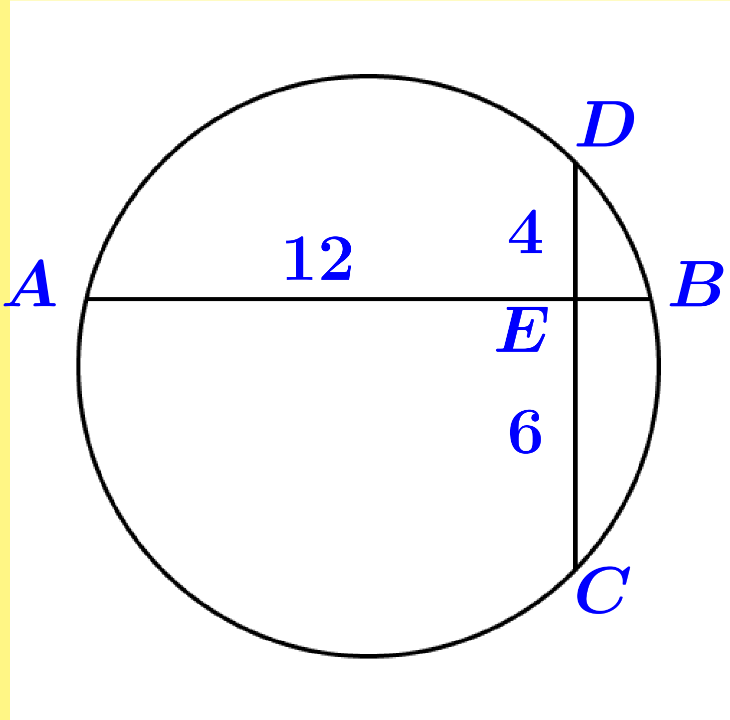
$\boldsymbol{N} = \mathbf{157}$ works, so the sum of its digits is **13**

Problem 26:



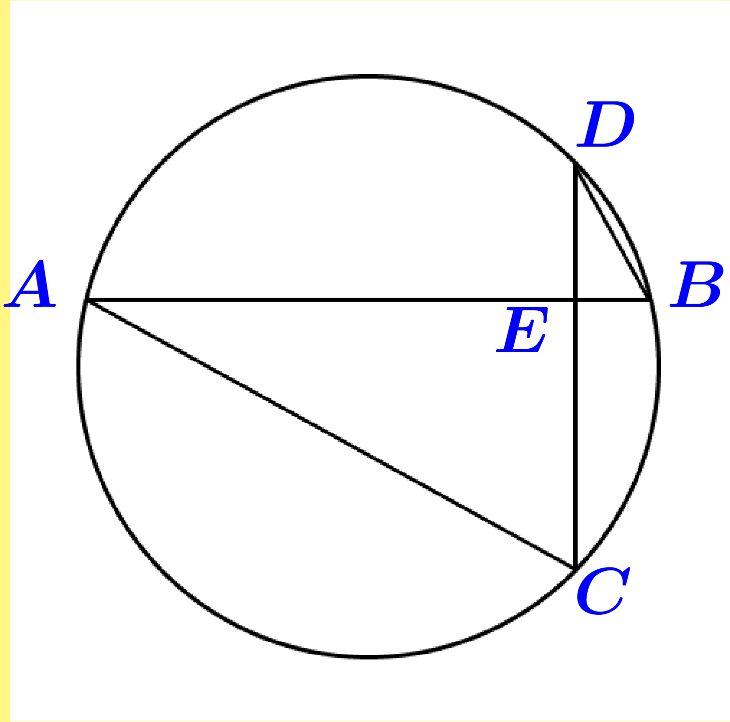
What’s the area of this circle?

Problem 26:



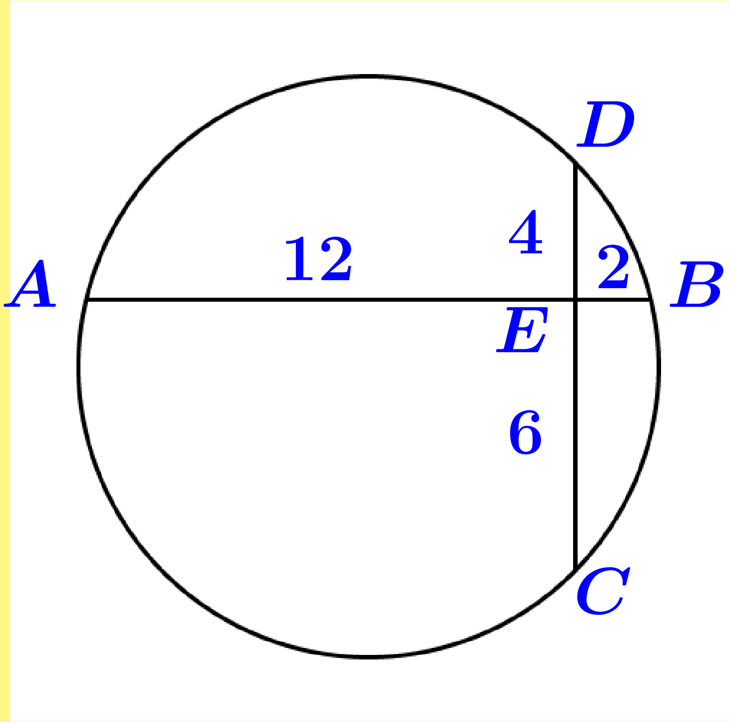
$$AE \cdot BE = CE \cdot DE$$

Problem 26:



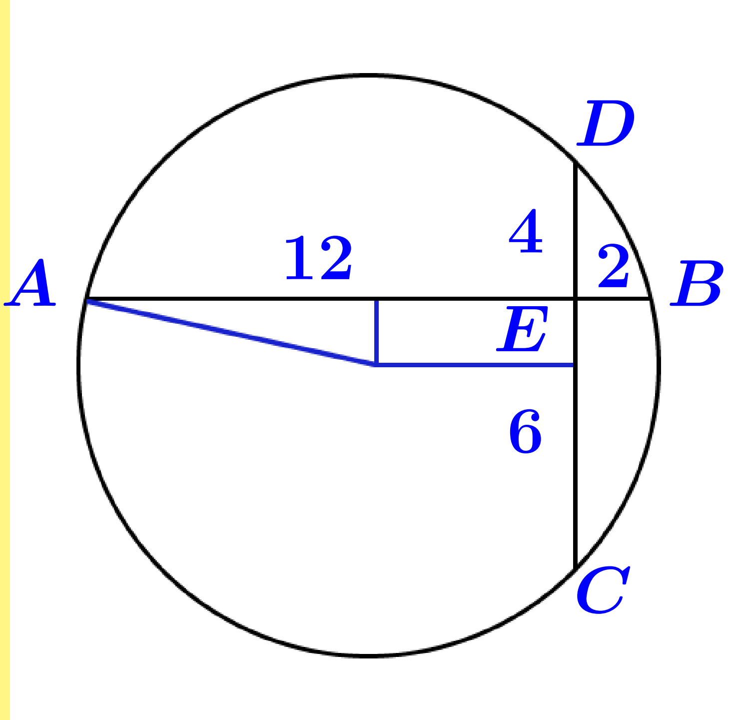
$$AE \cdot BE = CE \cdot DE$$

Problem 26:



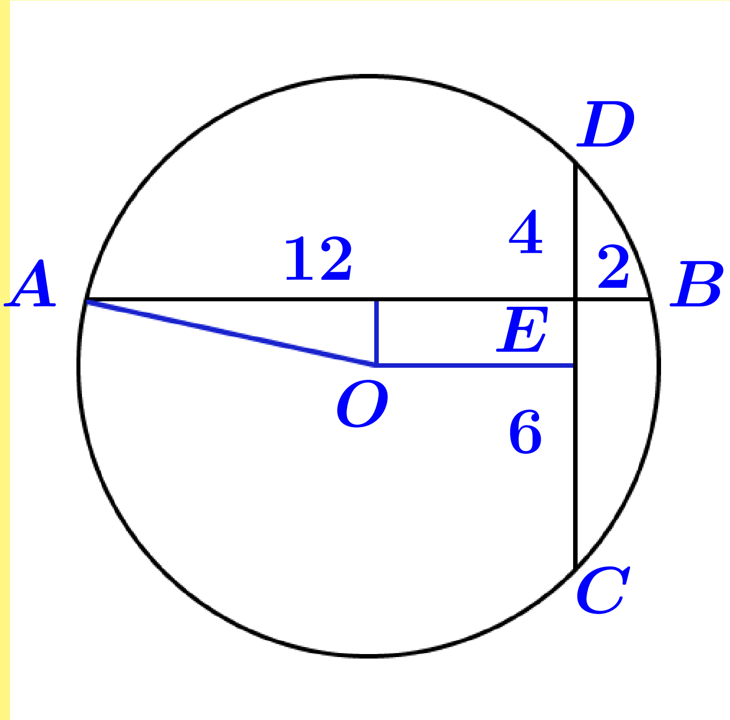
$$AE \cdot BE = CE \cdot DE \implies BE = 2$$

Problem 26:



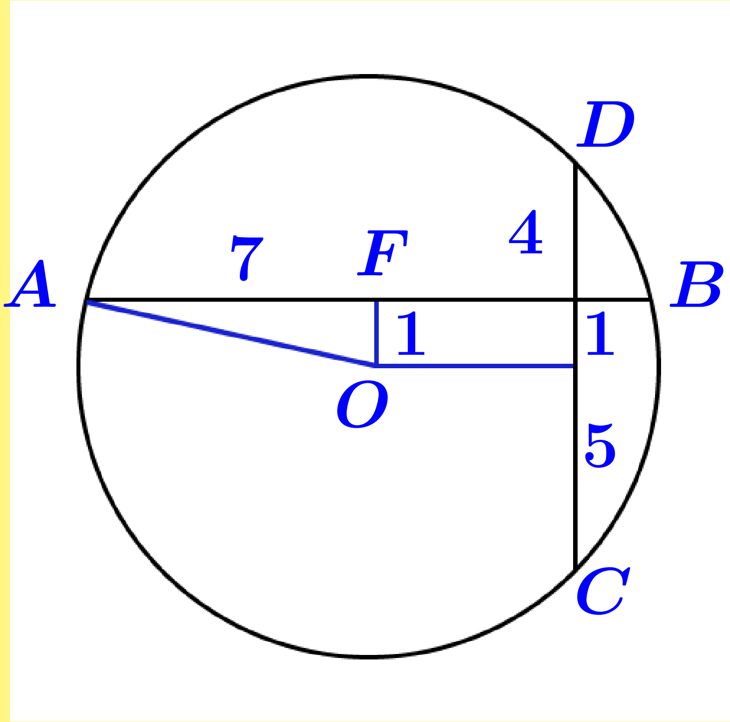
$$AE \cdot BE = CE \cdot DE \implies BE = 2$$

Problem 26:



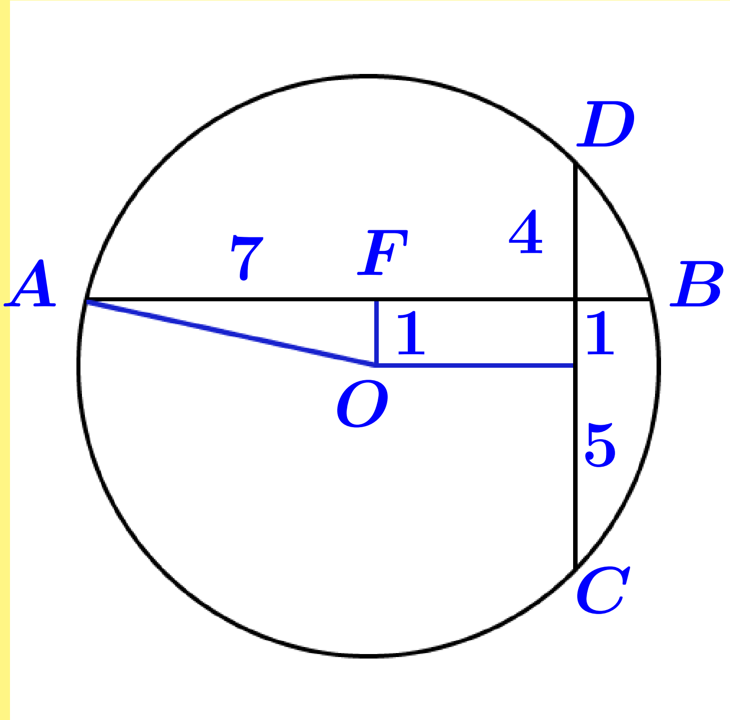
$$AE \cdot BE = CE \cdot DE \implies BE = 2$$

Problem 26:



$$AE \cdot BE = CE \cdot DE \implies BE = 2$$

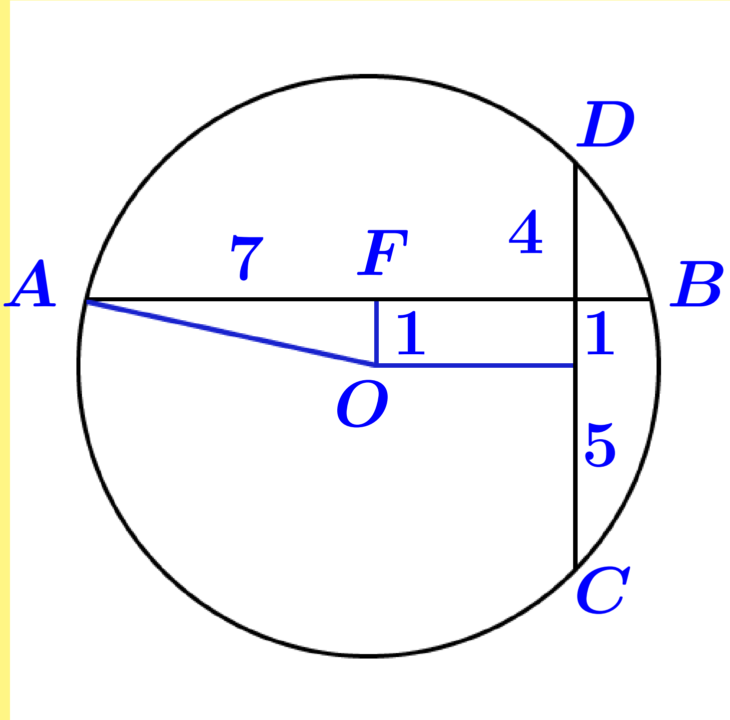
Problem 26:



$$AE \cdot BE = CE \cdot DE \implies BE = 2$$

$$0A^2 = 1^2 + 7^2$$

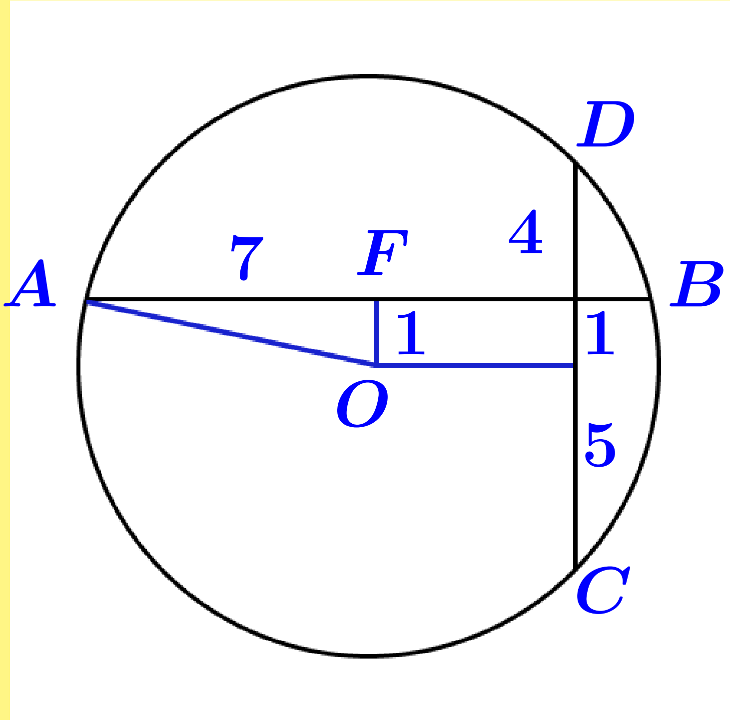
Problem 26:



$$AE \cdot BE = CE \cdot DE \implies BE = 2$$

$$0A^2 = 1^2 + 7^2 = 50$$

Problem 26:



$$AE \cdot BE = CE \cdot DE \implies BE = 2$$

$$0A^2 = 1^2 + 7^2 = 50 \implies \text{Area} = 50\pi$$

Problem 27:

When is $\mathbf{3^n + 81}$ a square?

Problem 27:

When is $3^n + 81$ a square?

$n = 1$

Problem 27:

When is $\mathbf{3^{n}+81}$ a square?

$$n = 1 \implies 3^{n} + 81 = 84$$

Problem 27:

When is $\mathbf{3^n + 81}$ a square?

$$\boldsymbol{n = 1 \implies 3^n + 81 = 84}$$
$$\boldsymbol{n = 2 \implies 3^n + 81 = 90}$$

Problem 27:

When is $3^n + 81$ a square?

$$
\begin{aligned}
n = 1 &\implies 3^n + 81 = 84 \\
n = 2 &\implies 3^n + 81 = 90 \\
n = 3 &\implies 3^n + 81 = 108
\end{aligned}
$$

Problem 27:

When is $3^n + 81$ a square?

$$n = 1 \implies 3^n + 81 = 84$$

$$n = 2 \implies 3^n + 81 = 90$$

$$n = 3 \implies 3^n + 81 = 108$$

$$n = 4 \implies 3^n + 81 = 162$$

Problem 27:

When is $\mathbf{3}^n + \mathbf{81}$ a square?

$$\begin{aligned} n = 1 &\implies 3^n + 81 = 84 \\ n = 2 &\implies 3^n + 81 = 90 \\ n = 3 &\implies 3^n + 81 = 108 \\ n = 4 &\implies 3^n + 81 = 162 \end{aligned}$$

$n = k + 4$ where k is a positive integer

Problem 27:

When is $3^n + 81$ a square?

$n = k + 4$ where k is a positive integer

Problem 27:

When is $\mathbf{3^{\boldsymbol{n}} + 81}$ a square?

$\boldsymbol{n = k + 4}$ where $\boldsymbol{k}$ is a positive integer

$$\mathbf{3^{\boldsymbol{n}} + 81}$$

Problem 27:

When is $3^n + 81$ a square?

$n = k + 4$ where k is a positive integer

$$3^n + 81 = 3^{k+4} + 81$$

Problem 27:

When is $\mathbf{3^n + 81}$ a square?

$\boldsymbol{n = k + 4}$ where $\boldsymbol{k}$ is a positive integer

$$\mathbf{3^n + 81 = 3^{k+4} + 81 = 81\left(3^k + 1\right)}$$

Problem 27:

When is $3^n + 81$ a square?

$n = k + 4$ where k is a positive integer

$$3^n + 81 = 3^{k+4} + 81 = 81\left(3^k + 1\right)$$

$$3^k + 1 = x^2$$

Problem 27:

When is $3^n + 81$ a square?

$n = k + 4$ where k is a positive integer

$$3^n + 81 = 3^{k+4} + 81 = 81\left(3^k + 1\right)$$

$$3^k + 1 = x^2 \implies 3^k = x^2 - 1$$

Problem 27:

When is $3^n + 81$ a square?

$n = k + 4$ where k is a positive integer

$$3^n + 81 = 3^{k+4} + 81 = 81\left(3^k + 1\right)$$

$$3^k + 1 = x^2 \implies 3^k = (x-1)(x+1)$$

Problem 27:

When is $3^n + 81$ a square?

$n = k + 4$ where k is a positive integer

$$3^n + 81 = 3^{k+4} + 81 = 81\left(3^k + 1\right)$$

$$3^k + 1 = x^2 \implies 3^k = (x-1)(x+1)$$

When are two consecutive odd numbers powers of 3?

Problem 27:

When is $3^n + 81$ a square?

$n = k + 4$ where k is a positive integer

$$3^n + 81 = 3^{k+4} + 81 = 81\left(3^k + 1\right)$$

$$3^k + 1 = x^2 \implies 3^k = (x-1)(x+1)$$

When are two consecutive odd numbers powers of 3?

$$x = 2$$

Problem 27:

When is $3^n + 81$ a square?

$n = k + 4$ where k is a positive integer

$$3^n + 81 = 3^{k+4} + 81 = 81\left(3^k + 1\right)$$

$$3^k + 1 = x^2 \implies 3^k = (x-1)(x+1)$$

When are two consecutive odd numbers powers of 3?

$$x = 2 \implies k = 1$$

Problem 27:

When is $\mathbf{3^n + 81}$ a square?

$n = k + 4$ where k is a positive integer

$$3^n + 81 = 3^{k+4} + 81 = 81\left(3^k + 1\right)$$

$$3^k + 1 = x^2 \implies 3^k = (x-1)(x+1)$$

When are two consecutive odd numbers powers of $\mathbf{3}$?

$$x = 2 \implies k = 1$$

Problem 27:

When is $\mathbf{3^n + 81}$ a square?

$\boldsymbol{n = k + 4}$ where $\boldsymbol{k}$ is a positive integer

$$\mathbf{3^n + 81 = 3^{k+4} + 81 = 81\left(3^k + 1\right)}$$

$$\mathbf{3^k + 1 = x^2 \implies 3^k = (x-1)(x+1)}$$

When are two consecutive odd numbers powers of $\mathbf{3}$?

$$\mathbf{x = 2 \implies k = 1 \implies n = 5}$$

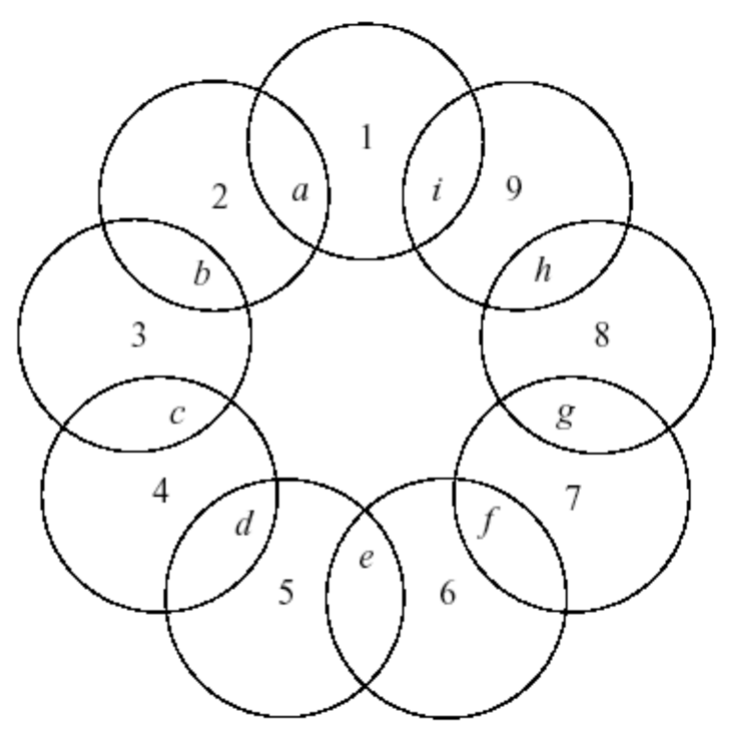
Problem 27:

When is $3^n + 81$ a square?

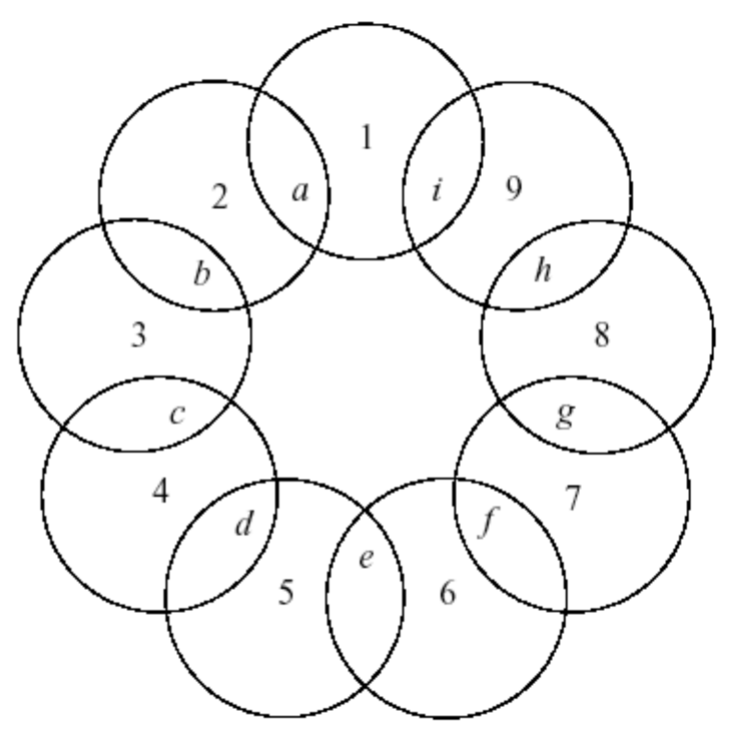
$$n = 5$$

There is 1 such n.

Problem 28:

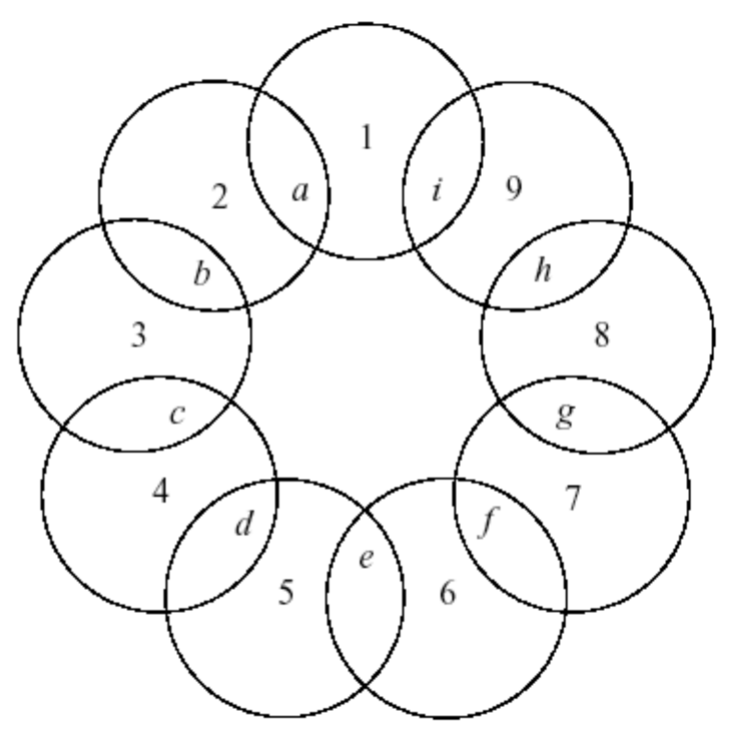


Problem 28:



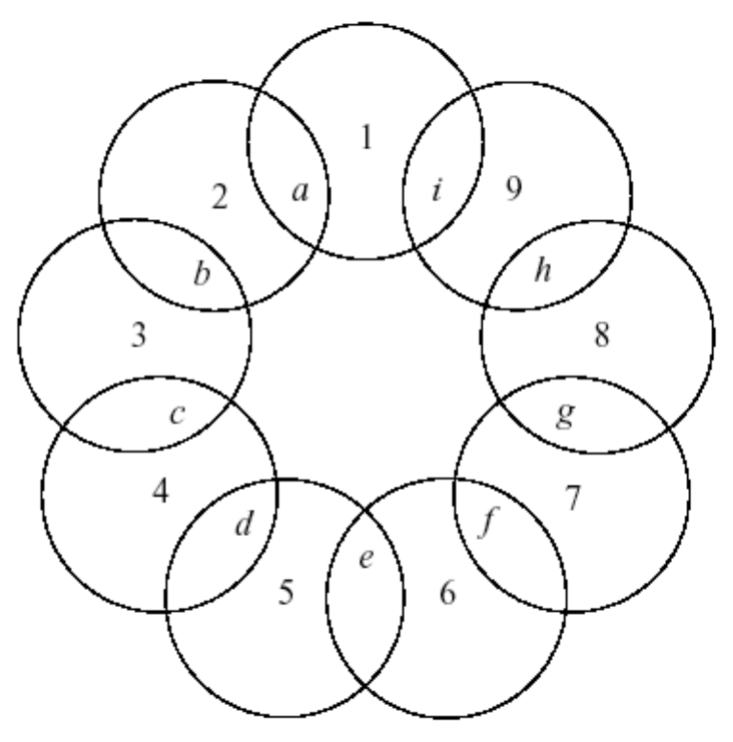
What’s $a + d + g$?

Problem 28:



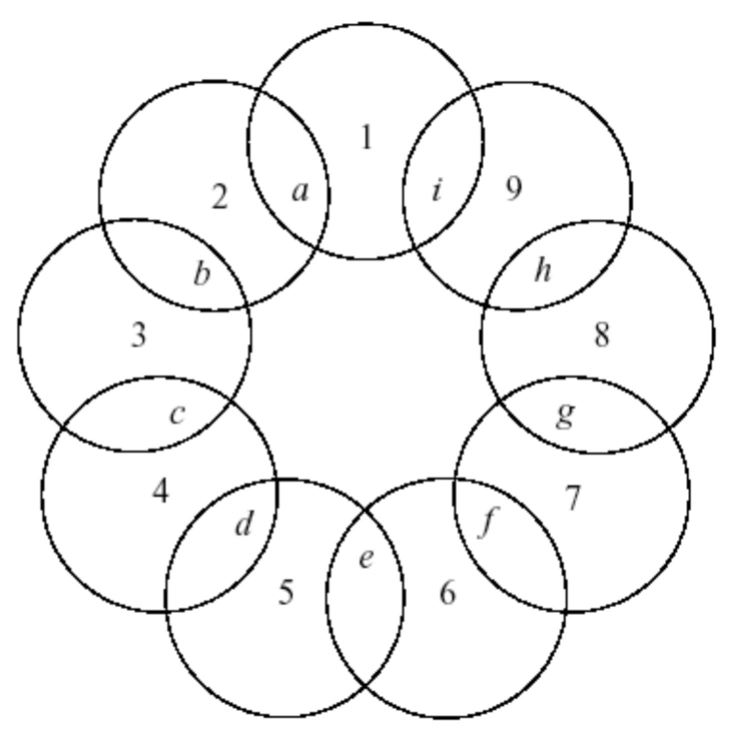
$$a + b + c + \cdots + i$$

Problem 28:



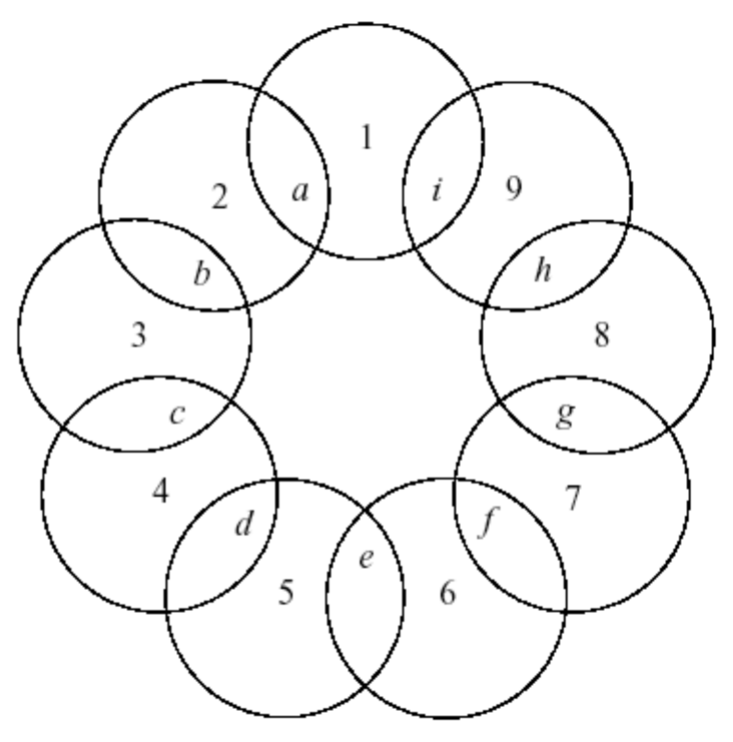
$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9$$

Problem 28:



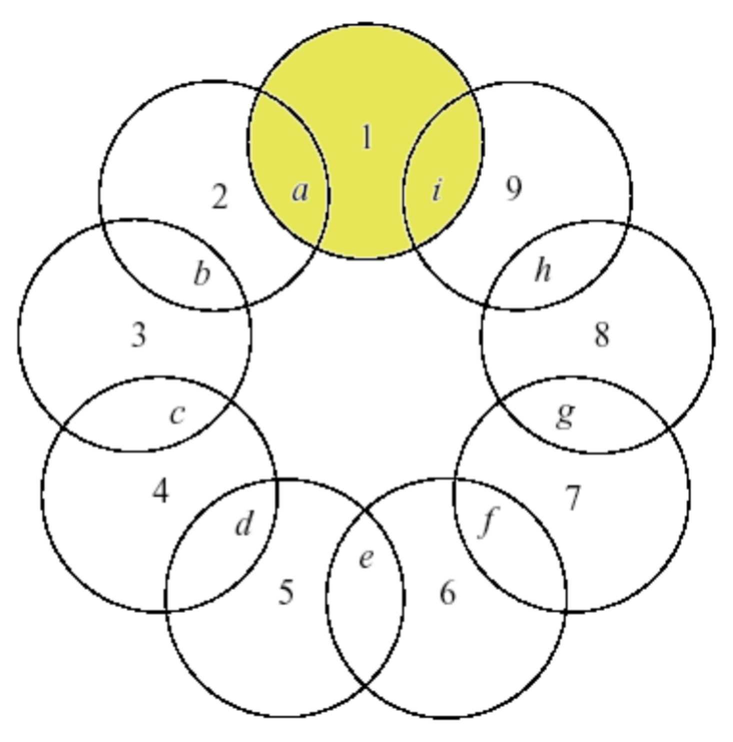
$$a+b+c+\cdots+i=1+2+3+\cdots+9=\frac{9\cdot 10}{2}$$

Problem 28:



$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

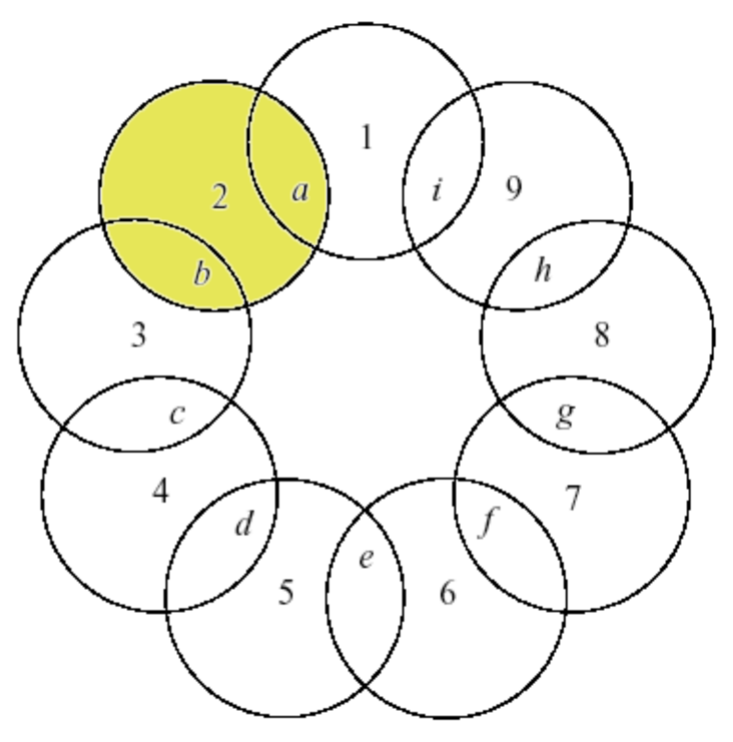
Problem 28:



$$a+b+c+\cdots+i=1+2+3+\cdots+9=45$$

$$S=i+a+1$$

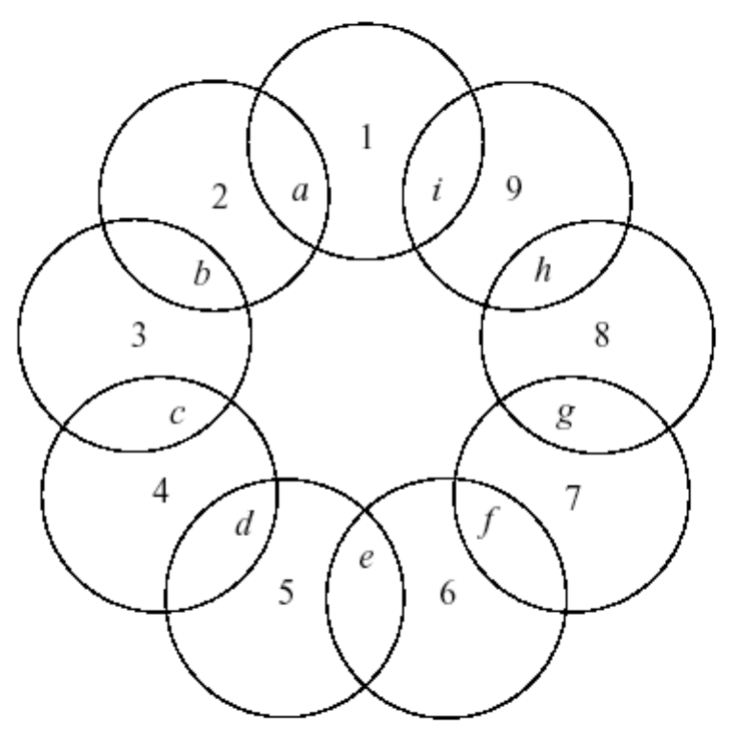
Problem 28:



$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

$$2S = i + a + 1 + a + b + 2$$

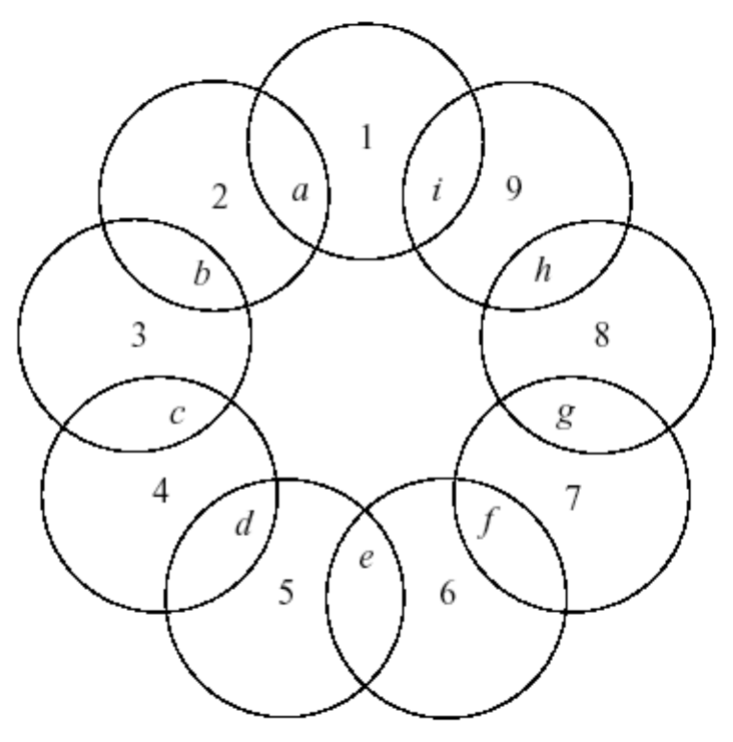
Problem 28:



$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

$$9S = i + a + 1 + a + b + 2 + b + c + 3 + \cdots$$

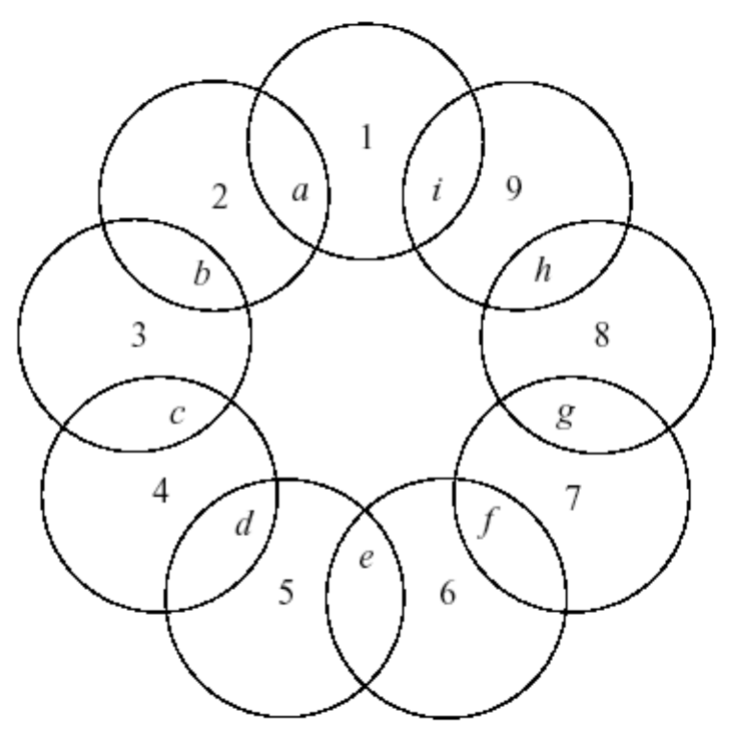
Problem 28:



$$a+b+c+\cdots+i=1+2+3+\cdots+9=45$$

$$9S=2(a+b+c+\cdots+i)+1+2+3+\cdots+9$$

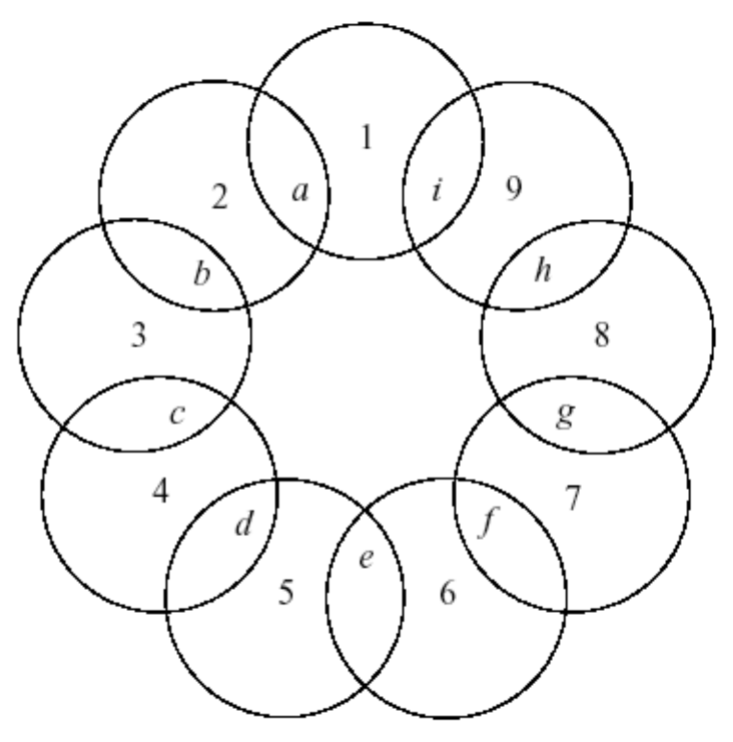
Problem 28:



$$a+b+c+\cdots+i=1+2+3+\cdots+9=45$$

$$9S=2(\underbrace{a+b+c+\cdots+i}_{45})+1+2+3+\cdots+9$$

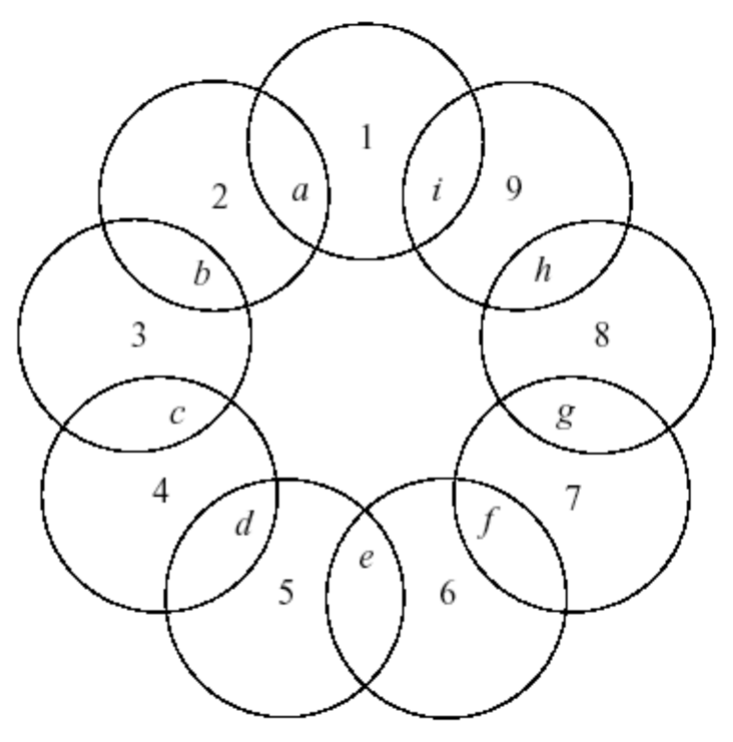
Problem 28:



$$a+b+c+\cdots+i=1+2+3+\cdots+9=45$$

$$9S=2(\underbrace{a+b+c+\cdots+i}_{45})+\underbrace{1+2+3+\cdots+9}_{45}$$

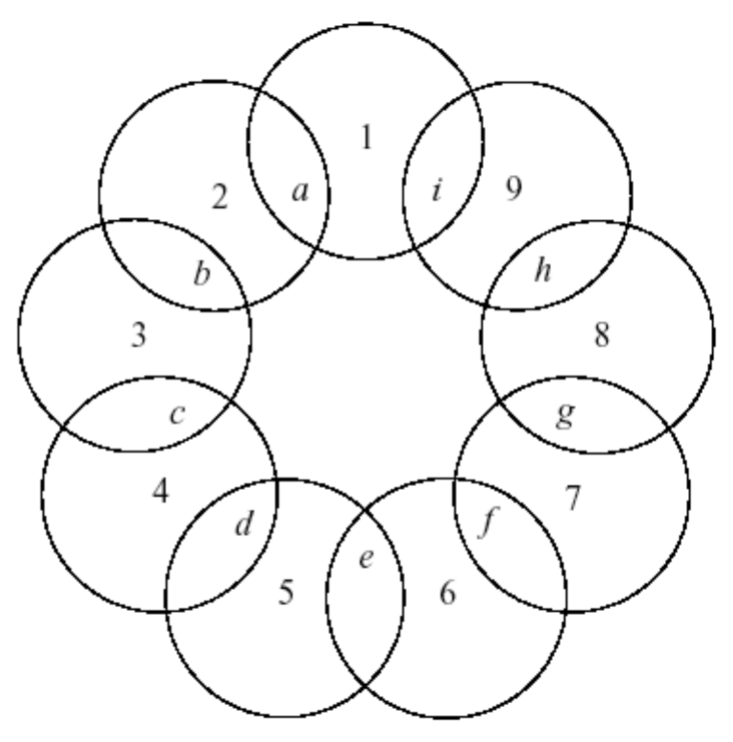
Problem 28:



$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

$$9S = 3 \times 45$$

Problem 28:

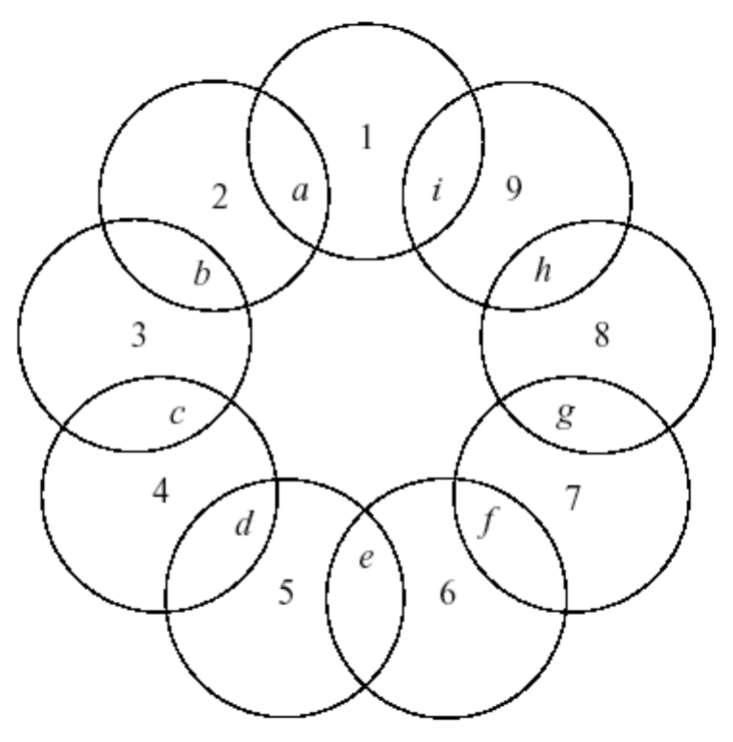


$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

$$9S = 3 \times 45 \implies S = 15$$

Problem 28:

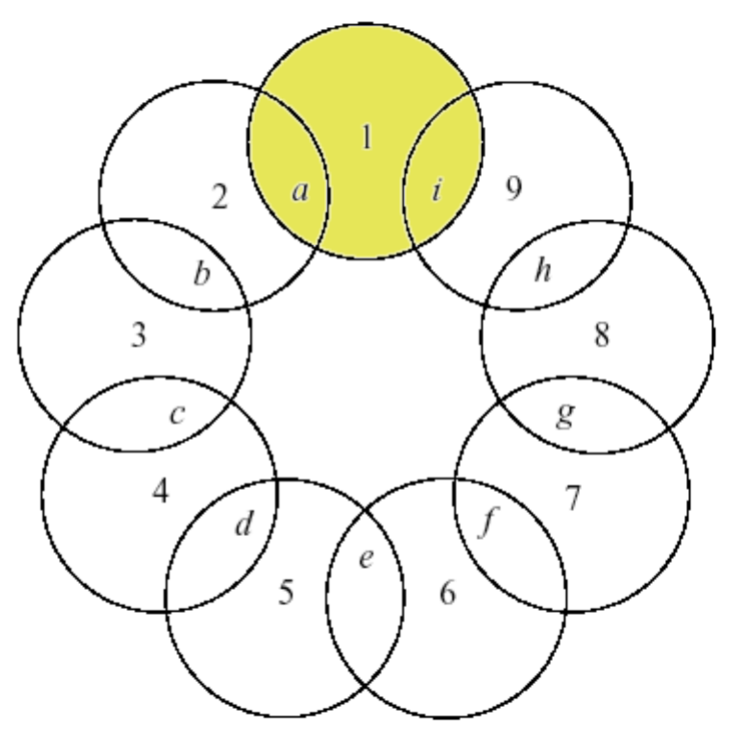
$S = 15$



$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

Problem 28:

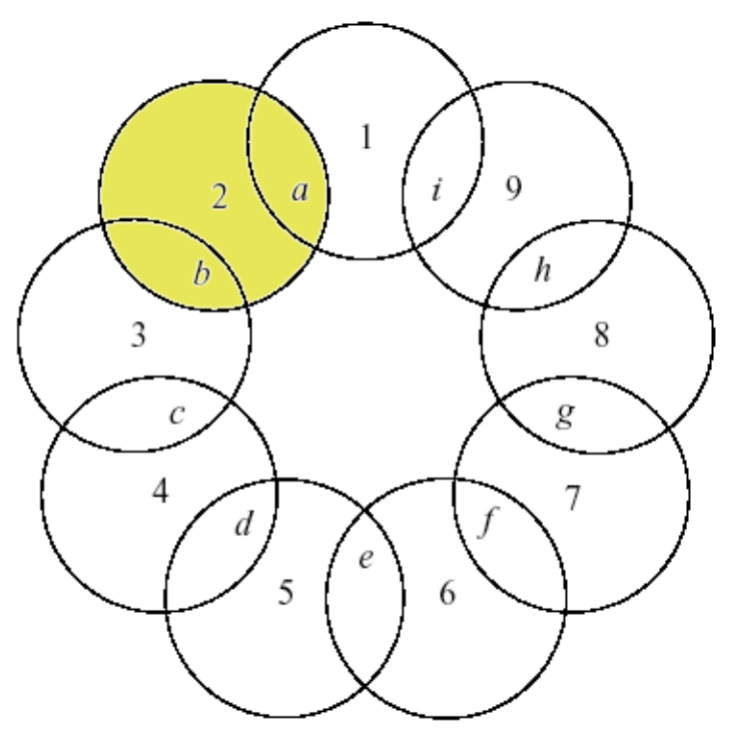
$S = 15$



$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

Problem 28:

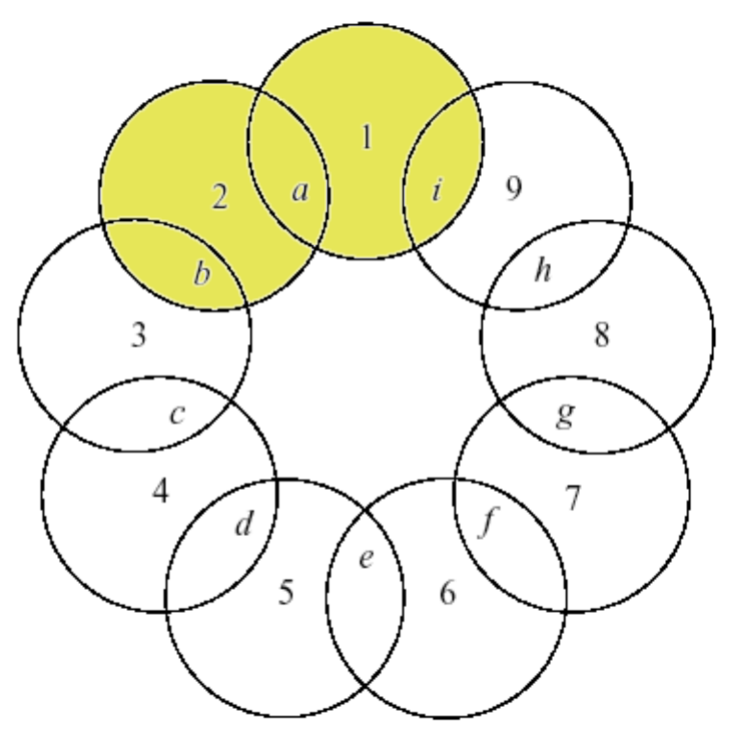
$S = 15$



$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

Problem 28:

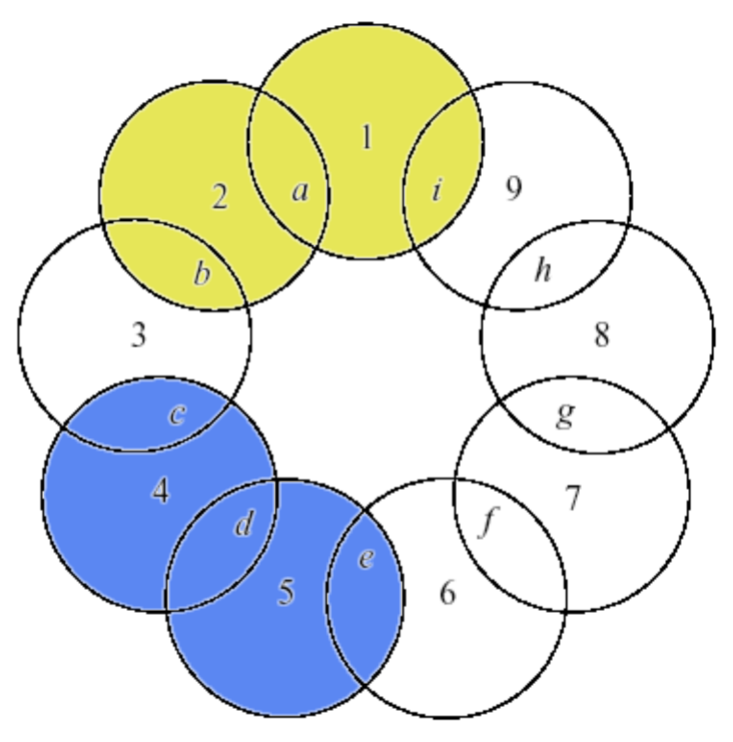
$S = 15$



$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

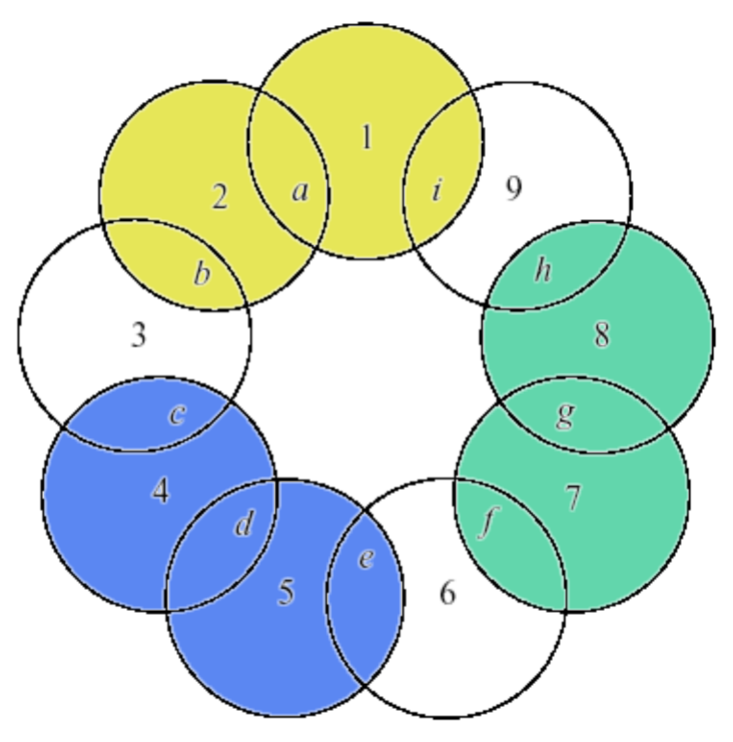
Problem 28:

$S = 15$



$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

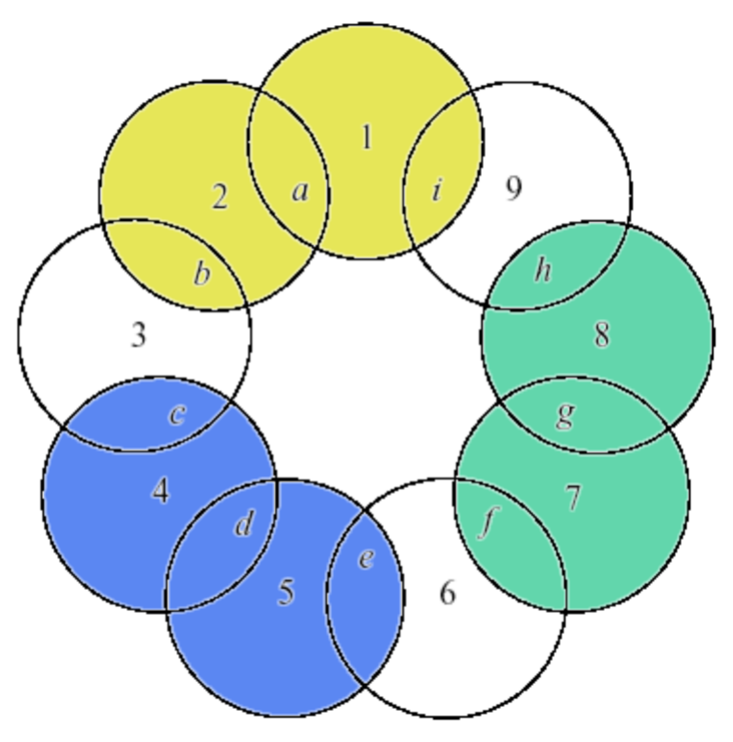
Problem 28: $S = 15$



$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

Problem 28:

$S = 15$

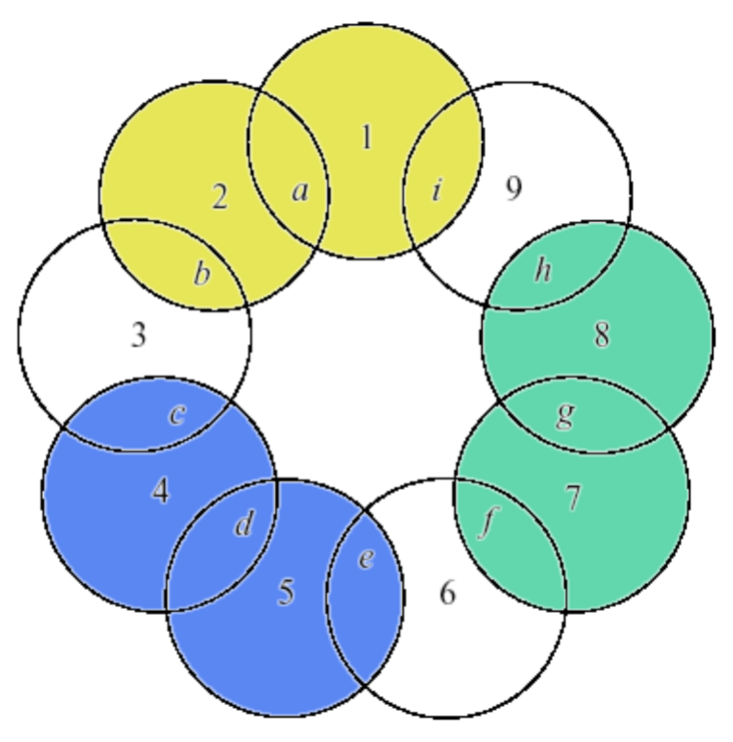


$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

$$6S = 27 + 45 + a + d + g$$

Problem 28:

$S = 15$

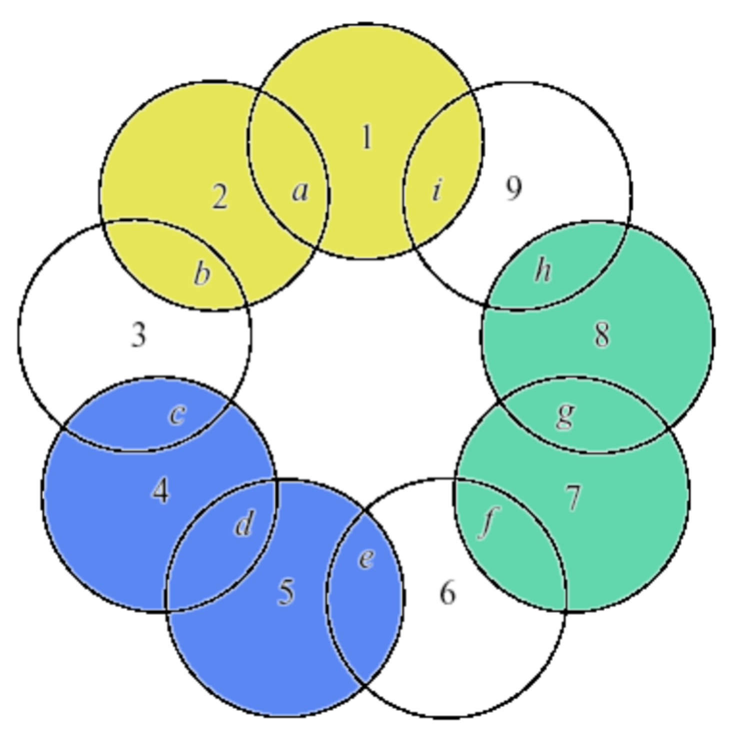


$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

$$6S = 27 + 45 + a + d + g = 72 + a + d + g$$

Problem 28:

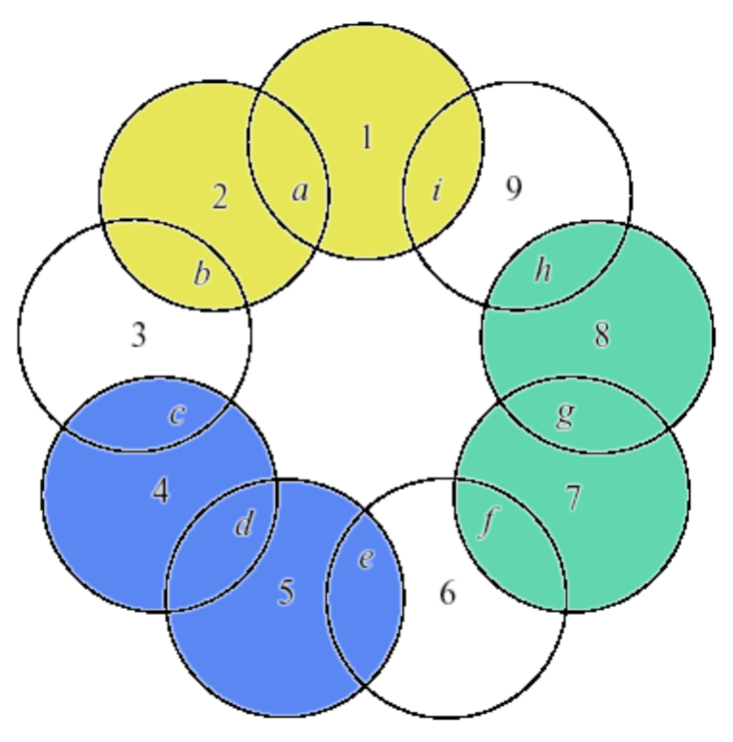
$S = 15$



$$a + b + c + \cdots + i = 1 + 2 + 3 + \cdots + 9 = 45$$

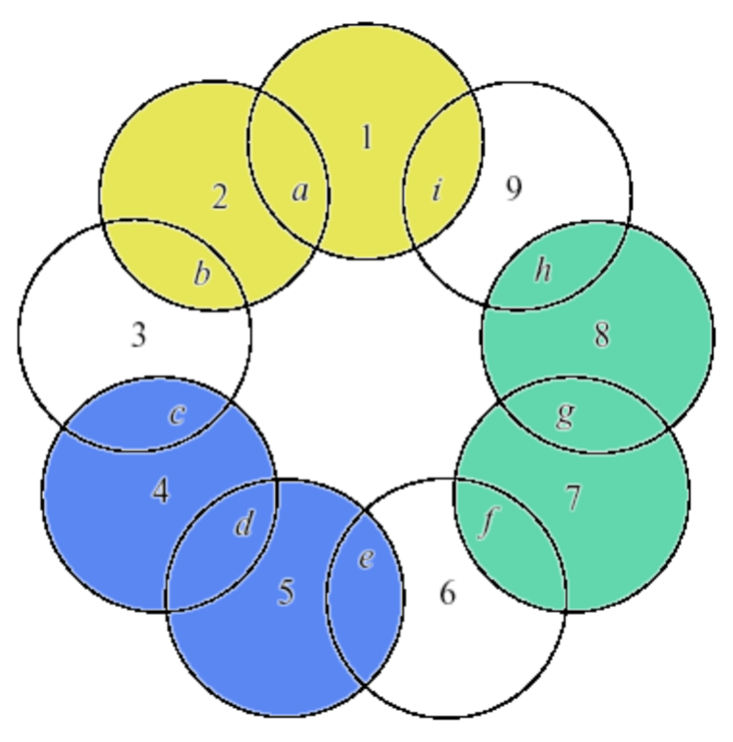
$$90 = 6S = 27 + 45 + a + d + g = 72 + a + d + g$$

Problem 28:



$$a + d + g = 18$$

Problem 28:



$$a + d + g = 18$$

Problem 29:

What is the smallest positive integer $\boldsymbol{n}$ such that $\mathbf{31}$ divides $\mathbf{5}^{\boldsymbol{n}} + \boldsymbol{n}$?

Problem 29:

What is the smallest positive integer $\boldsymbol{n}$
such that $\mathbf{31}$ divides $\mathbf{5}^{\boldsymbol{n}} + \boldsymbol{n}$?

$$\mathbf{5^3 - 1}$$

Problem 29:

What is the smallest positive integer $\boldsymbol{n}$ such that $\mathbf{31}$ divides $\mathbf{5}^{\boldsymbol{n}} + \boldsymbol{n}$?

$$\mathbf{5^3 - 1 = (5 - 1)(5^2 + 5 + 1)}$$

Problem 29:

What is the smallest positive integer n
such that 31 divides $5^n + n$?

$$5^3 - 1 = (5 - 1)(5^2 + 5 + 1) = 4 \cdot 31$$

Problem 29:

What is the smallest positive integer n
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$$5^3 - 1 = (5 - 1)(5^2 + 5 + 1) = 4 \cdot 31$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3q + r$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3\underset{\substack{\uparrow \\ \text{quotient}}}{q} + r$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3\underset{\underset{\text{quotient}}{\uparrow}}{q} + \underset{\underset{\underset{\text{remainder}}{\vert}}{\uparrow}}{r}$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3\underset{\substack{\uparrow \\ \text{quotient}}}{q} + \underset{\substack{\uparrow \\ | \\ \text{remainder}}}{r} \qquad r \in \{0, 1, 2\}$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3q + r \qquad r \in \{0, 1, 2\}$$

$$5^n - 5^r$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3q + r \qquad r \in \{0, 1, 2\}$$

$$5^n - 5^r = 5^{3q+r} - 5^r$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3q + r \qquad r \in \{0, 1, 2\}$$

$$5^n - 5^r = 5^{3q+r} - 5^r = 5^r\left(5^{3q} - 1\right)$$

Problem 29:

What is the smallest positive integer n
such that 31 divides $5^n + n$?

$$n = 3q + r \qquad r \in \{0, 1, 2\}$$

$$5^n - 5^r = 5^{3q+r} - 5^r = 5^r(5^3 - 1)k$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3q + r \qquad r \in \{0, 1, 2\}$$

$$5^n - 5^r = 5^{3q+r} - 5^r = 5^r \cdot 4 \cdot 31 \cdot k$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3q + r \qquad r \in \{0, 1, 2\}$$

$$5^n - 5^r = 5^{3q+r} - 5^r = 5^r \cdot 4 \cdot 31 \cdot k$$

$$n + 5^r = (5^n + n) - (5^n - 5^r)$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3q + r \qquad r \in \{0, 1, 2\}$$

$$5^n - 5^r = 5^{3q+r} - 5^r = 5^r \cdot 4 \cdot 31 \cdot k$$

$$n + 5^r = (5^n + n) - (5^n - 5^r) = 31 \cdot k'$$

Problem 29:

What is the smallest positive integer n
such that 31 divides $5^n + n$?

$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

$$r = 0$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

$$r = 0 \implies n + 1 = 31 \cdot k'$$

Problem 29:

What is the smallest positive integer n
such that 31 divides $5^n + n$?

$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

$$r = 0 \implies n + 1 = 31 \cdot k' \quad \implies n = 30$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

$$r = 0 \implies n + 1 = 31 \cdot k' \implies n = 30$$

$$r = 1$$

Problem 29:

What is the smallest positive integer $\boldsymbol{n}$
such that $\mathbf{31}$ divides $\mathbf{5}^{\boldsymbol{n}} + \boldsymbol{n}$?

$$\boldsymbol{n = 3q + r} \quad \boldsymbol{r \in \{0, 1, 2\}} \quad \boldsymbol{n + 5^r = 31 \cdot k'}$$

$$\boldsymbol{r = 0 \implies n + 1 = 31 \cdot k' \quad \implies n = 30}$$

$$\boldsymbol{r = 1 \implies n + 5 = 31 \cdot k'}$$

Problem 29:

What is the smallest positive integer n
such that 31 divides $5^n + n$?

$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

$$r = 0 \implies n + 1 = 31 \cdot k' \implies n = 30$$

$$r = 1 \implies n + 5 = 31 \cdot k' \implies n = 26$$

Problem 29:

What is the smallest positive integer n
such that 31 divides $5^n + n$?

$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

$$r = 0 \implies n + 1 = 31 \cdot k' \implies n = 30$$

$$r = 1 \implies n + 5 = 31 \cdot k' \implies n \neq 26$$

Problem 29:

What is the smallest positive integer n
such that 31 divides $5^n + n$?

$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

$$r = 0 \implies n + 1 = 31 \cdot k' \implies n = 30$$

$$r = 1 \implies n + 5 = 31 \cdot k' \implies n = 88$$

Problem 29:

What is the smallest positive integer n
such that 31 divides $5^n + n$?

$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

$$r = 0 \implies n + 1 = 31 \cdot k' \implies n = 30$$

$$r = 1 \implies n + 5 = 31 \cdot k' \implies n = 88$$

$$r = 2$$

Problem 29:

What is the smallest positive integer n
such that 31 divides $5^n + n$?

$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

$$r = 0 \implies n + 1 = 31 \cdot k' \implies n = 30$$

$$r = 1 \implies n + 5 = 31 \cdot k' \implies n = 88$$

$$r = 2 \implies n + 25 = 31 \cdot k'$$

Problem 29:

What is the smallest positive integer n
such that 31 divides $5^n + n$?

$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

$$\begin{aligned} r = 0 &\implies n + 1 = 31 \cdot k' &&\implies n = 30 \\ r = 1 &\implies n + 5 = 31 \cdot k' &&\implies n = 88 \\ r = 2 &\implies n + 25 = 31 \cdot k' &&\implies n = 68 \end{aligned}$$

Problem 29:

What is the smallest positive integer n such that 31 divides $5^n + n$?

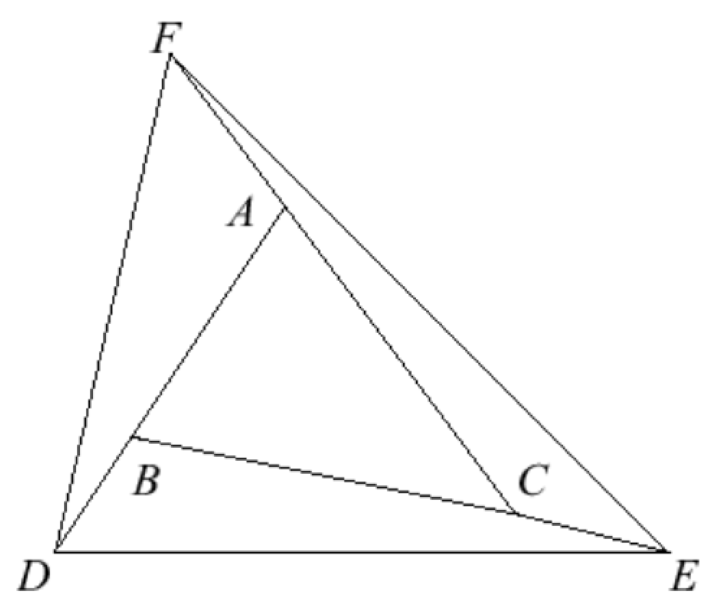
$$n = 3q + r \quad r \in \{0, 1, 2\} \quad n + 5^r = 31 \cdot k'$$

$$r = 0 \implies n + 1 = 31 \cdot k' \implies n = 30$$

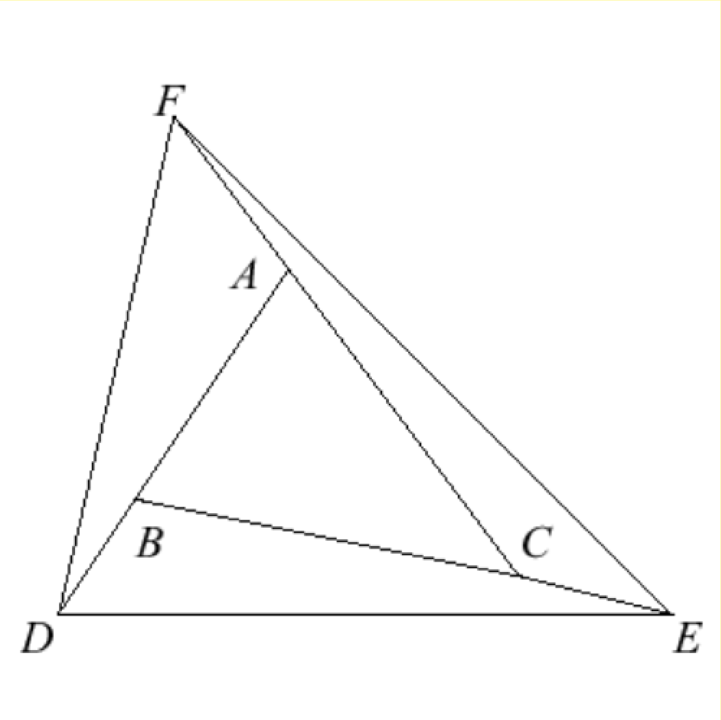
$$r = 1 \implies n + 5 = 31 \cdot k' \implies n = 88$$

$$r = 2 \implies n + 25 = 31 \cdot k' \implies n = 68$$

Problem 30:

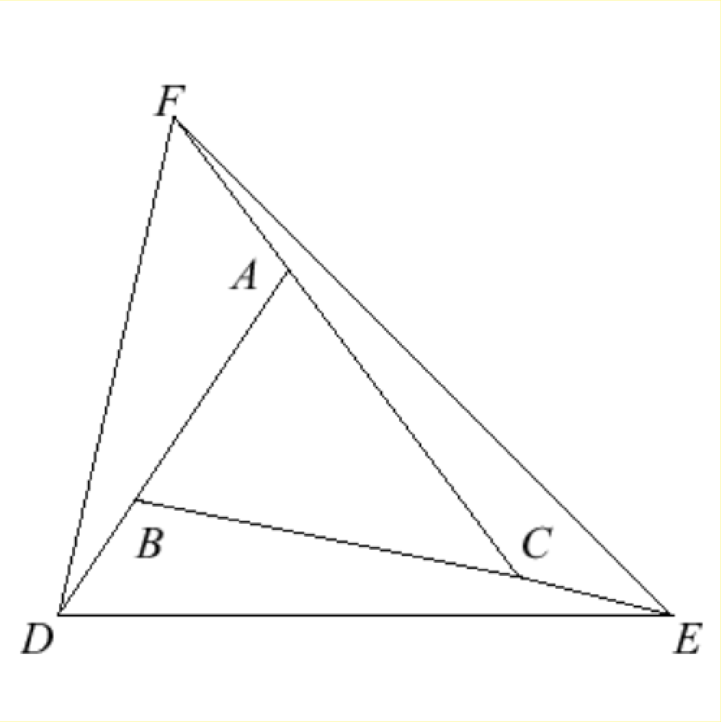


Problem 30:



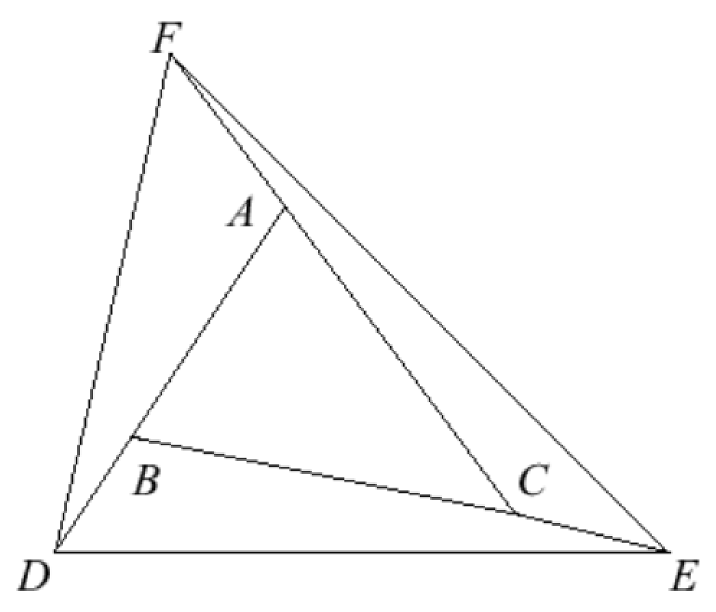
$$BD = \frac{1}{2}AB$$

Problem 30:



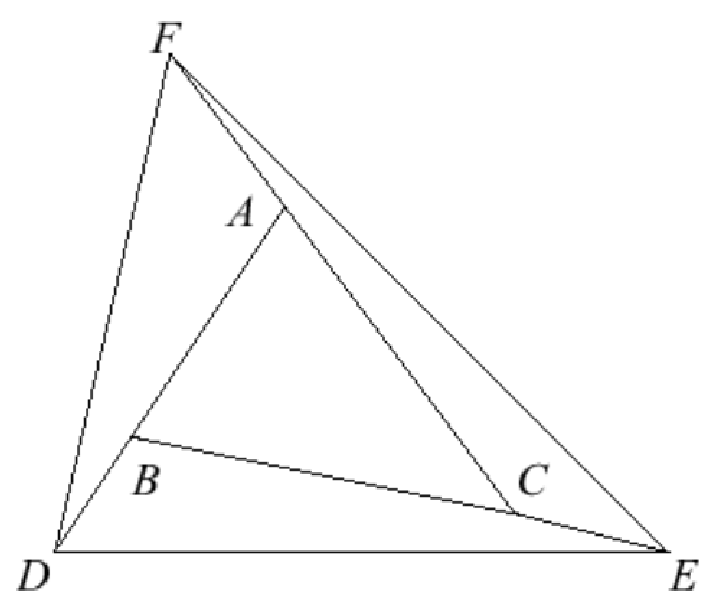
$$BD = \frac{1}{2}AB \qquad CE = \frac{1}{2}BC$$

Problem 30:



$$BD = \frac{1}{2}AB \qquad CE = \frac{1}{2}BC \qquad AF = \frac{1}{2}CA$$

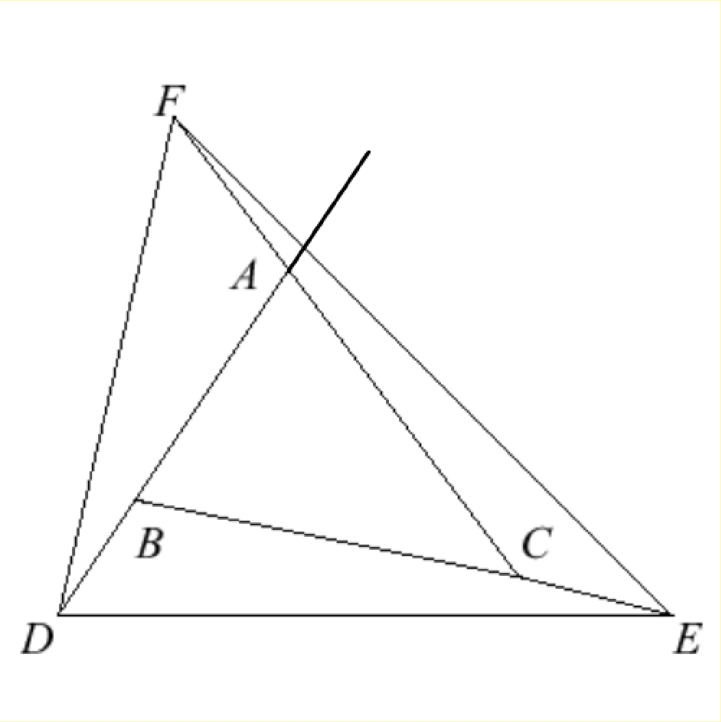
Problem 30:



$$BD = \frac{1}{2}AB \qquad CE = \frac{1}{2}BC \qquad AF = \frac{1}{2}CA$$

$$\frac{\text{area of } \triangle DEF}{\text{area of } \triangle ABC} = \quad ?$$

Problem 30:



$$BD = \frac{1}{2}AB \qquad CE = \frac{1}{2}BC \qquad AF = \frac{1}{2}CA$$

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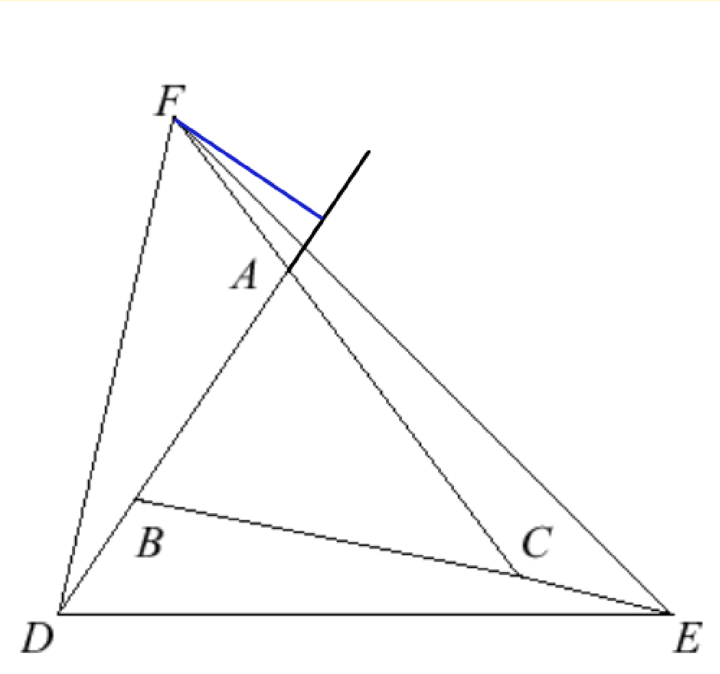
Problem 30:



$$BD = \frac{1}{2}AB \qquad CE = \frac{1}{2}BC \qquad AF = \frac{1}{2}CA$$

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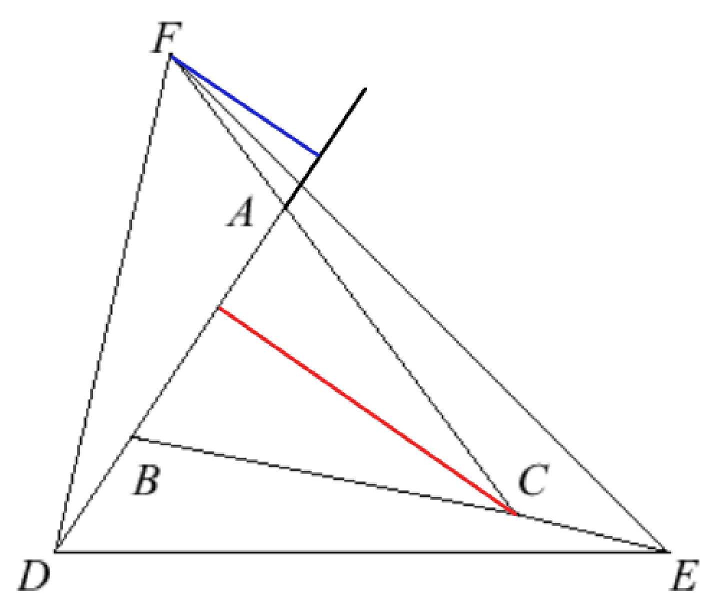
Problem 30:



$$BD = \frac{1}{2}AB \qquad CE = \frac{1}{2}BC \qquad AF = \frac{1}{2}CA$$

$$\frac{\text{area of } \triangle DEF}{\text{area of } \triangle ABC} = \quad ?$$

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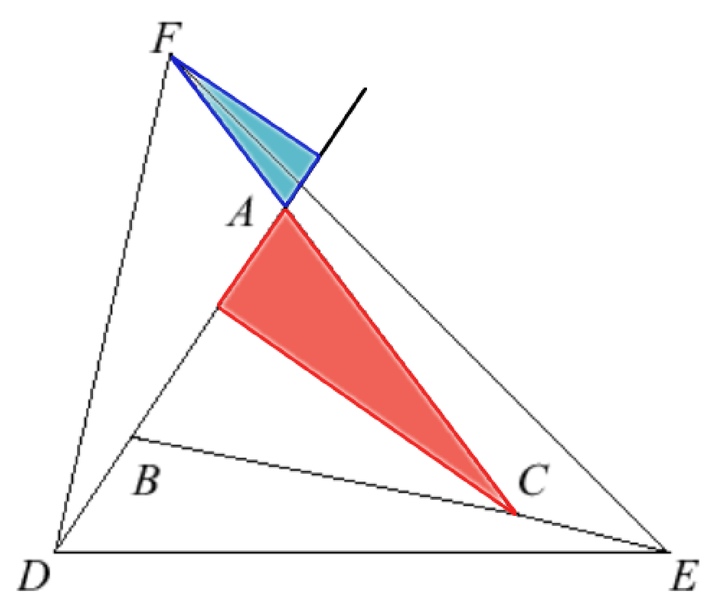
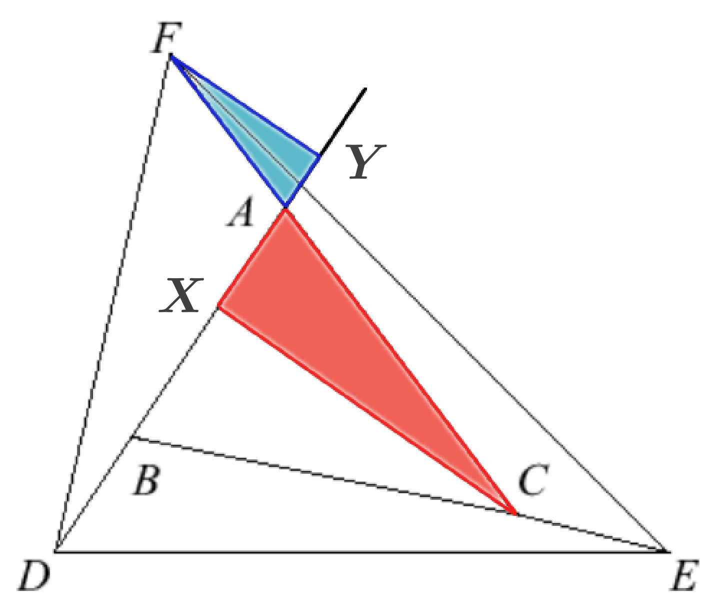


$$BD = \frac{1}{2}AB \qquad CE = \frac{1}{2}BC \qquad AF = \frac{1}{2}CA$$

$$\frac{\text{area of } \triangle DEF}{\text{area of } \triangle ABC} = \quad ?$$

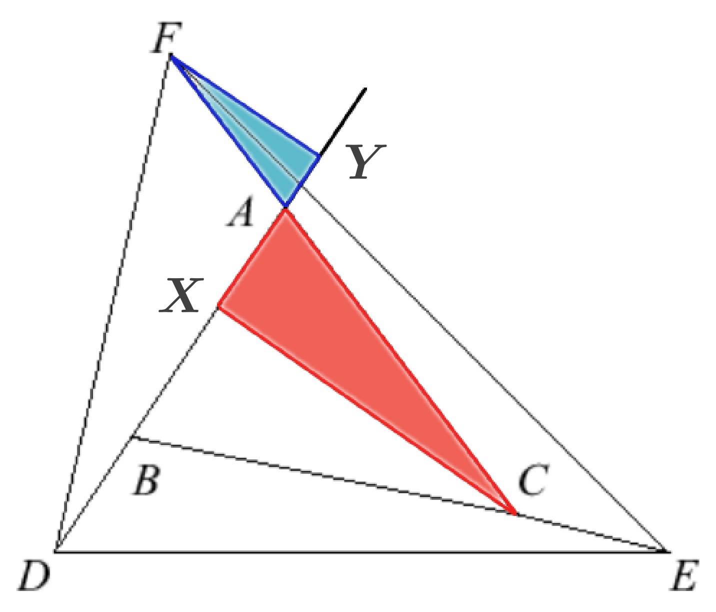
Problem 30:



$$BD = \frac{1}{2}AB \qquad CE = \frac{1}{2}BC \qquad AF = \frac{1}{2}CA$$

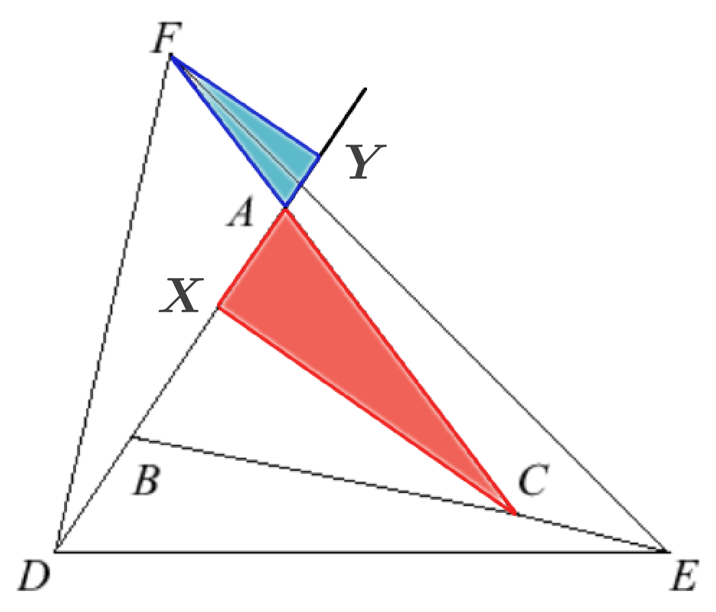
$$\frac{\text{area of } \triangle DEF}{\text{area of } \triangle ABC} = \quad ?$$

Problem 30:



$\triangle AXC$ is similar to $\triangle AYF$

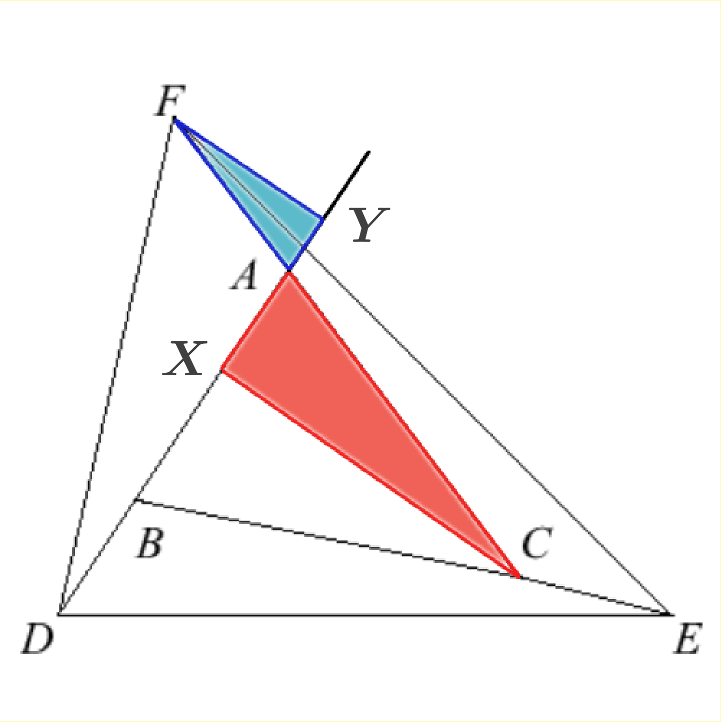
Problem 30:



$\triangle AXC$ is similar to $\triangle AYF$

$$\frac{FY}{CX} = \frac{AF}{AC}$$

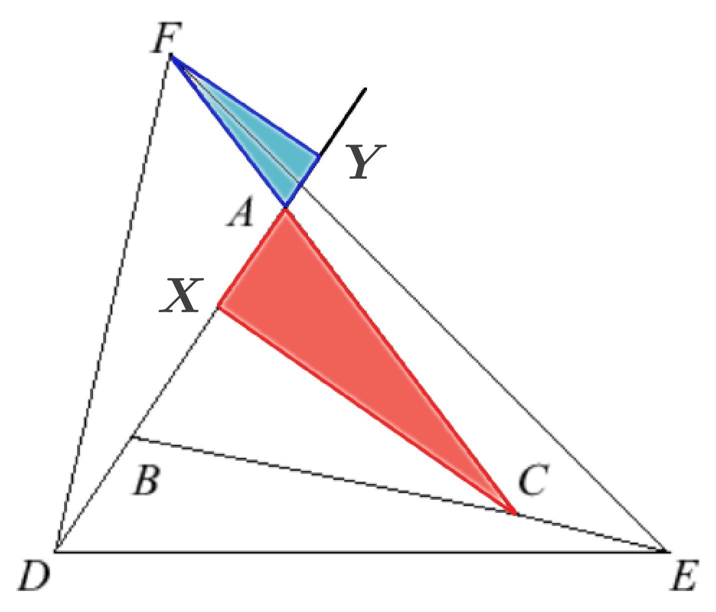
Problem 30:



$\triangle AXC$ is similar to $\triangle AYF$

$$\frac{FY}{CX} = \frac{AF}{AC} = \frac{1}{2}$$

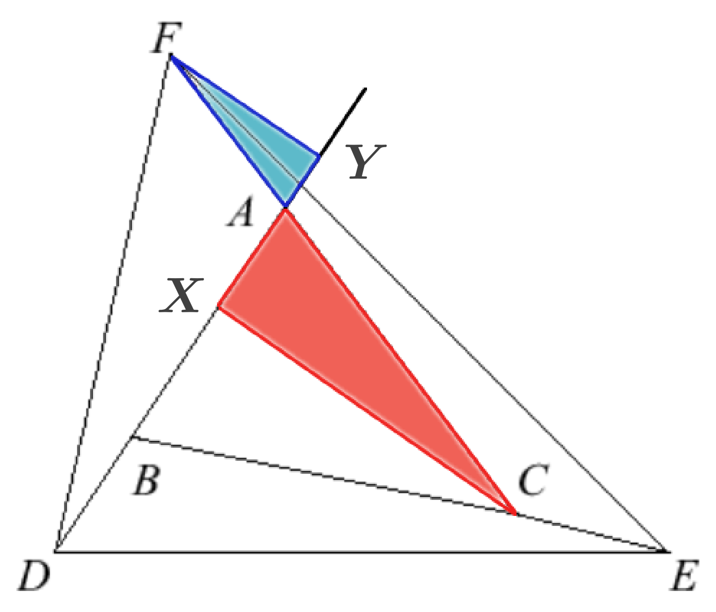
Problem 30:



$\triangle AXC$ is similar to $\triangle AYF$

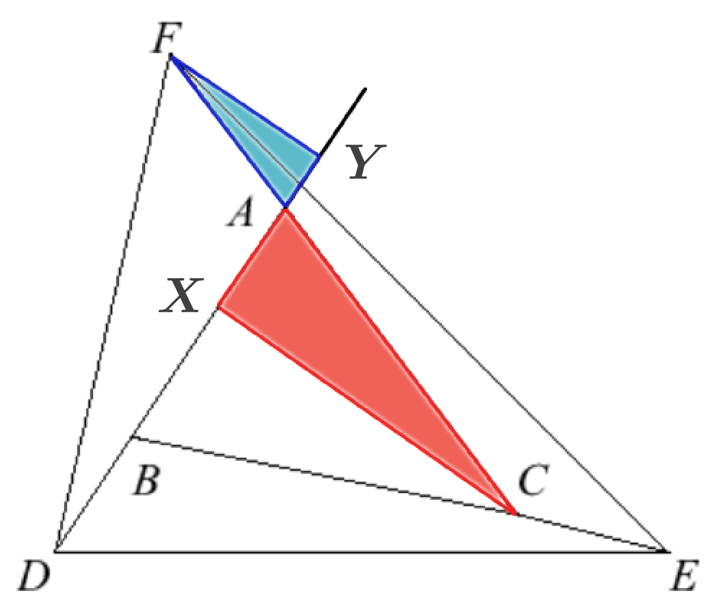
$$\frac{FY}{CX} = \frac{AF}{AC} = \frac{1}{2} \implies FY = \frac{1}{2}CX$$

Problem 30:



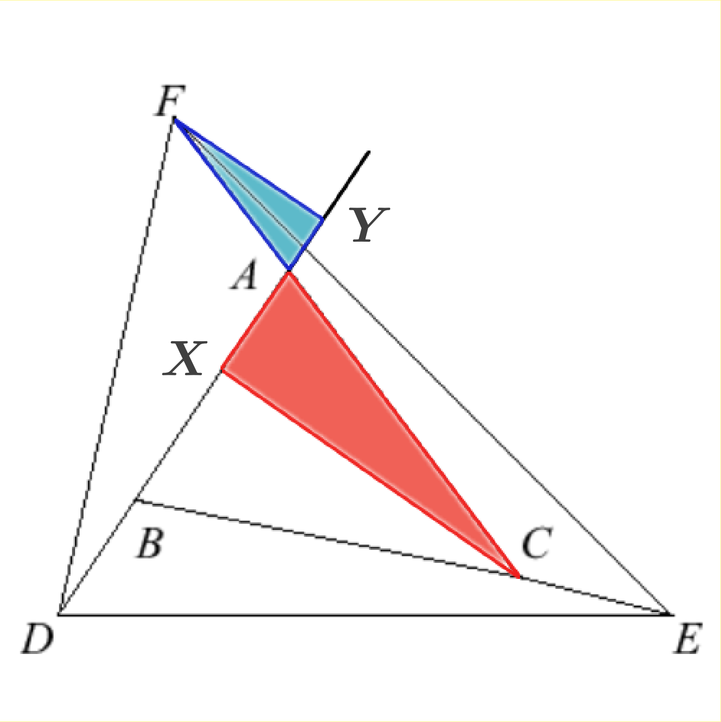
$$AD = ?\,AB \qquad FY = \frac{1}{2}CX$$

Problem 30:



$$AD = \frac{3}{2}AB \qquad FY = \frac{1}{2}CX$$

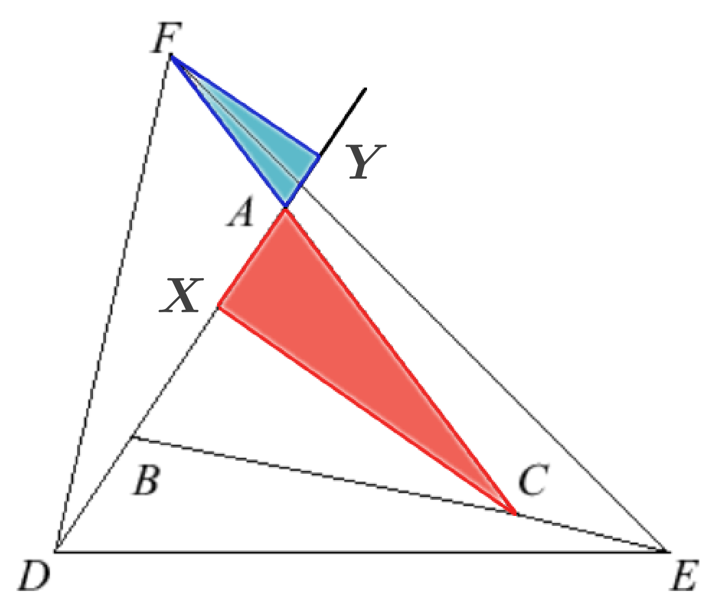
Problem 30:



$$AD = \frac{3}{2}AB \qquad FY = \frac{1}{2}CX$$

$$\mathcal{A}(\triangle ADF) =$$

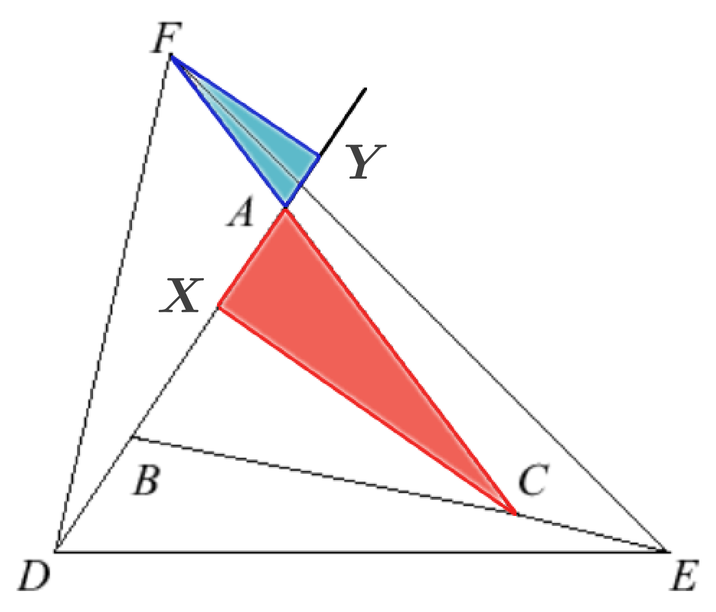
Problem 30:



$$AD = \frac{3}{2}AB \qquad FY = \frac{1}{2}CX$$

$$\mathcal{A}(\triangle ADF) = \frac{1}{2} \cdot \ AD \cdot FY$$

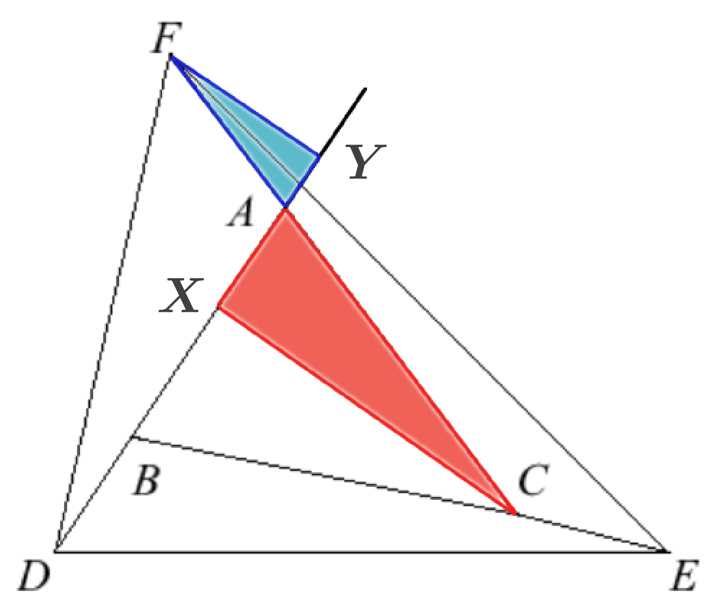
Problem 30:



$$AD = \frac{3}{2}AB \qquad FY = \frac{1}{2}CX$$

$$\mathcal{A}(\triangle ADF) = \frac{1}{2} \cdot \frac{3}{2}AB \cdot FY$$

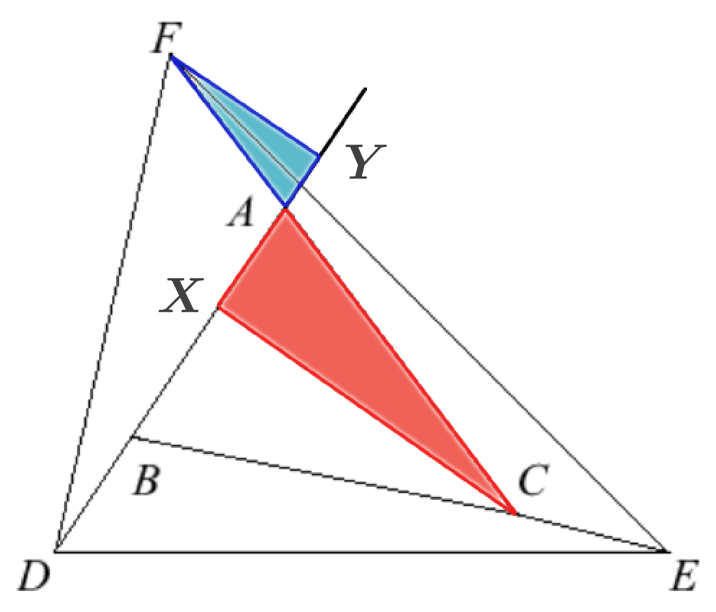
Problem 30:



$$AD = \frac{3}{2}AB \qquad FY = \frac{1}{2}CX$$

$$\mathcal{A}(\triangle ADF) = \frac{1}{2} \cdot \frac{3}{2}AB \cdot \frac{1}{2}CX$$

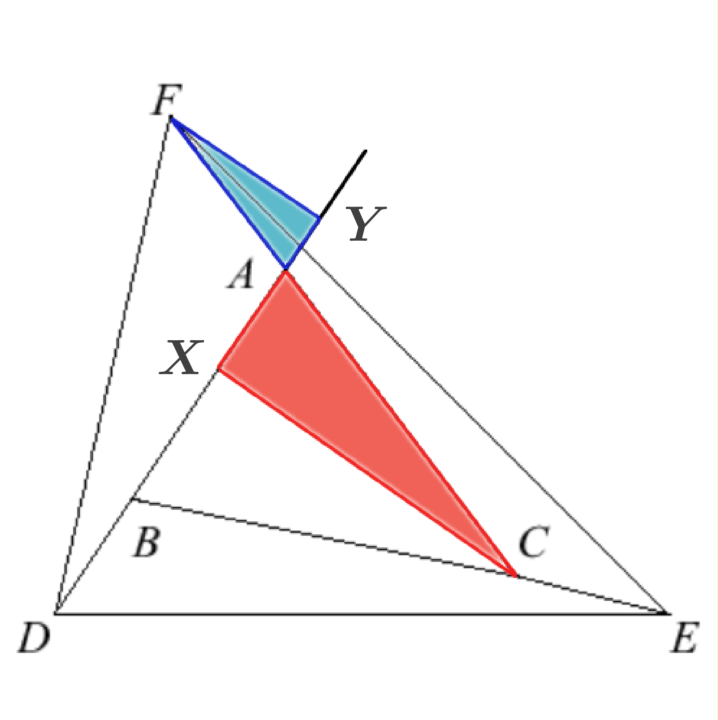
Problem 30:



$$AD = \frac{3}{2}AB \qquad FY = \frac{1}{2}CX$$

$$\mathcal{A}(\triangle ADF) = \frac{3}{4}\left(\frac{1}{2}\cdot AB \cdot CX\right)$$

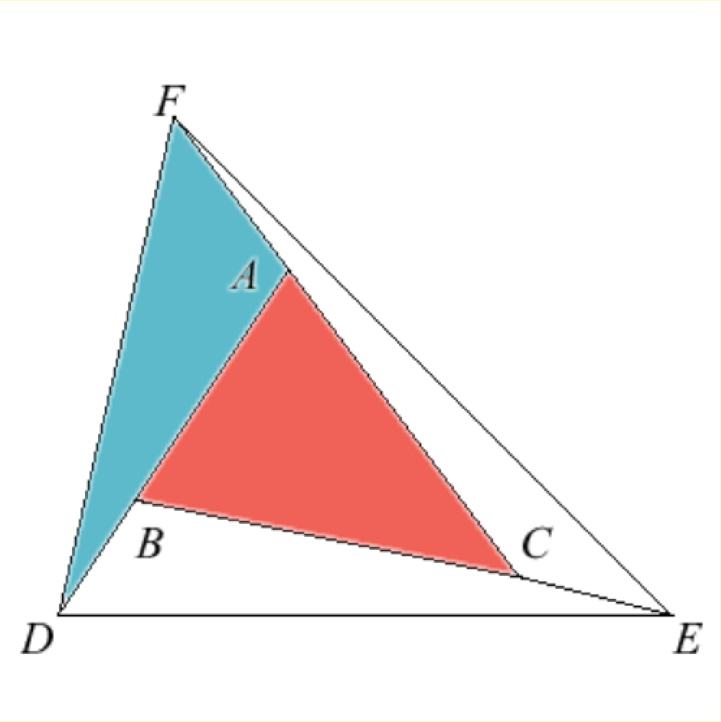
Problem 30:



$$AD = \frac{3}{2}AB \qquad FY = \frac{1}{2}CX$$

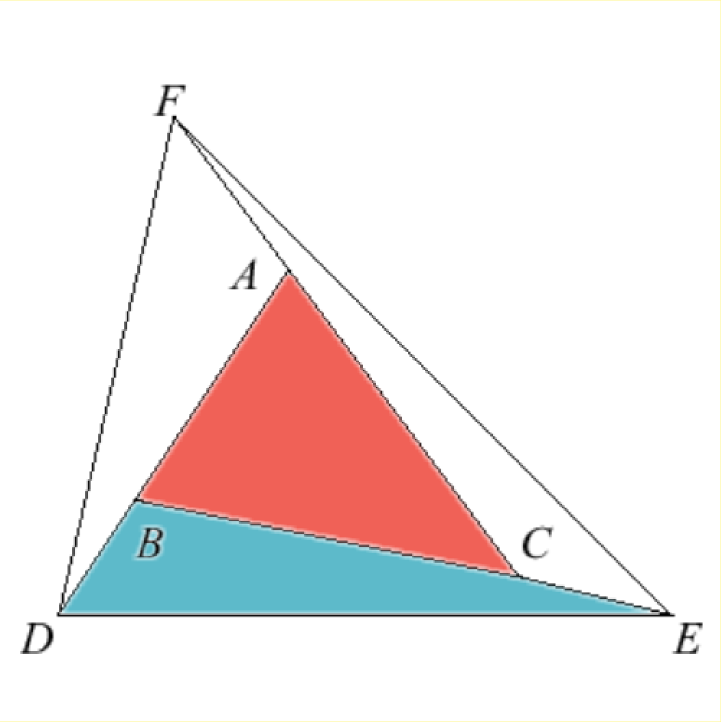
$$\mathcal{A}(\triangle ADF) = \frac{3}{4} \cdot \mathcal{A}(\triangle ABC)$$

Problem 30:



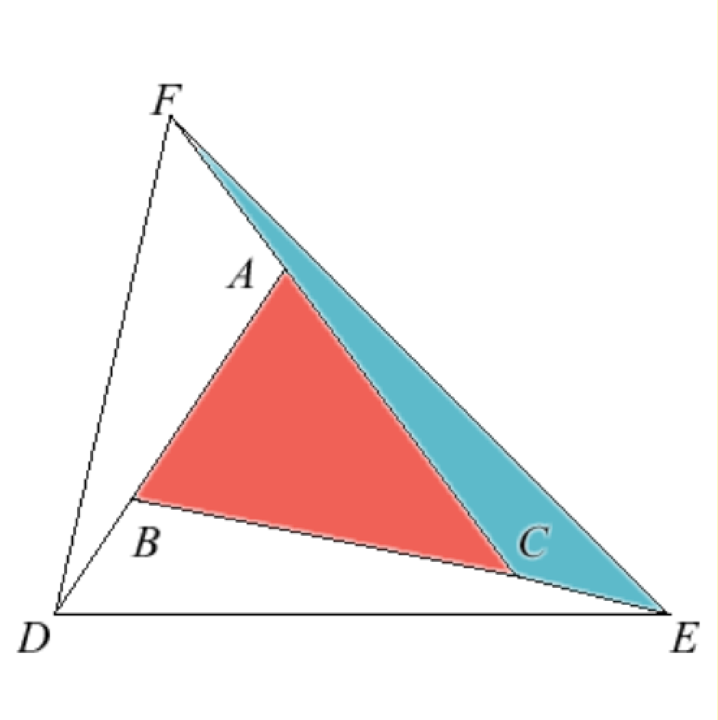
$$\mathcal{A}(\triangle ADF) = (3/4)\mathcal{A}(\triangle ABC)$$

Problem 30:



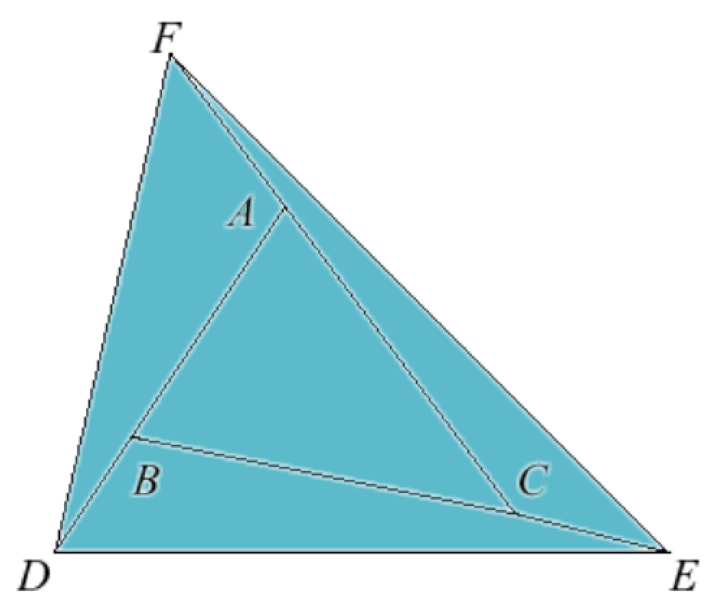
$$\mathcal{A}(\triangle ADF) = (3/4)\mathcal{A}(\triangle ABC)$$
$$\mathcal{A}(\triangle BED) = (3/4)\mathcal{A}(\triangle ABC)$$

Problem 30:



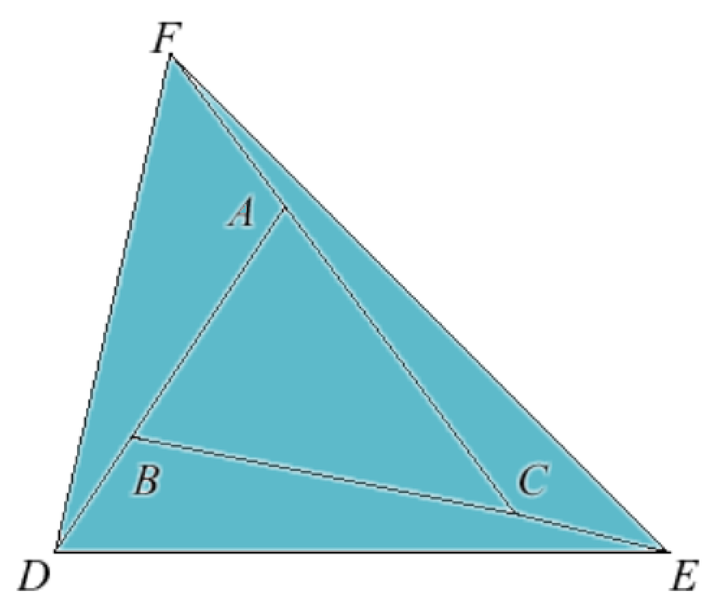
$$\mathcal{A}(\triangle ADF) = (3/4)\mathcal{A}(\triangle ABC)$$
$$\mathcal{A}(\triangle BED) = (3/4)\mathcal{A}(\triangle ABC)$$
$$\mathcal{A}(\triangle CFE) = (3/4)\mathcal{A}(\triangle ABC)$$

Problem 30:



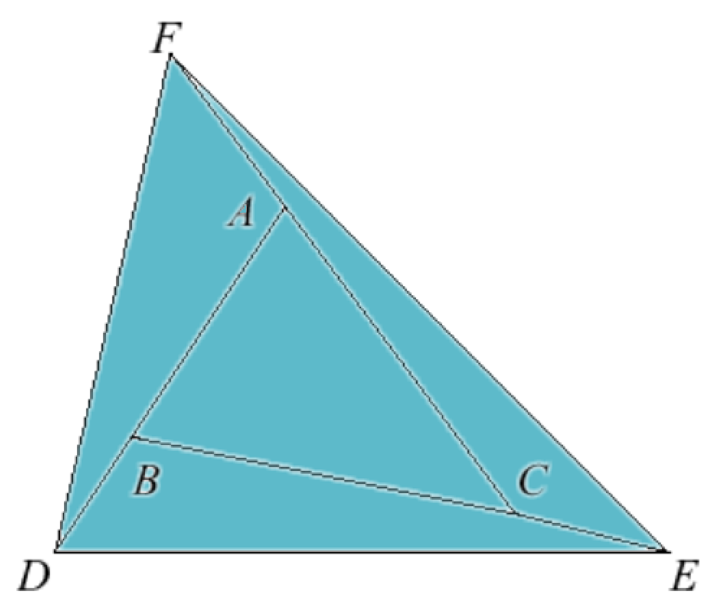
$$\mathcal{A}(\triangle DEF) = \left(1 + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}\right)\mathcal{A}(\triangle ABC)$$

Problem 30:



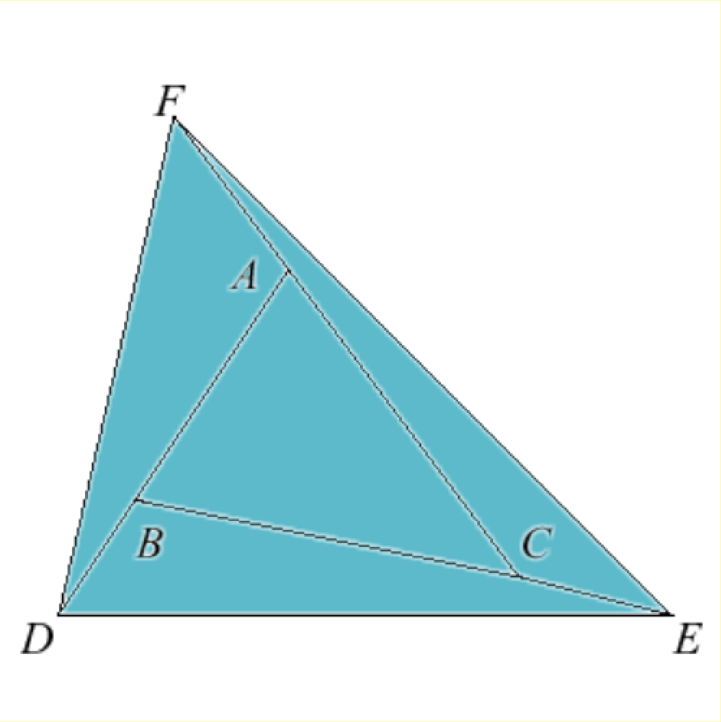
$$\mathcal{A}(\triangle DEF) = \left(\qquad \frac{13}{4} \qquad \right) \mathcal{A}(\triangle ABC)$$

Problem 30:



$$\mathcal{A}(\triangle DEF) = \frac{13}{4} A(\triangle ABC)$$

Problem 30:



$$\mathcal{A}(\triangle DEF) = \frac{13}{4} A(\triangle ABC)$$