THE DISTRIBUTION OF FRACTIONAL PARTS WITH APPLICATIONS TO GAP RESULTS IN NUMBER THEORY

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ABSTRACT: Let δ and N be positive real numbers, and let $f : \mathbb{R} \mapsto \mathbb{R}$ be any function. This paper obtains two estimates for the size of the set $\{u \in (N, 2N] : ||f(u)|| < \delta\}$, where u represents an integer and $||f(u)||$ represents the distance from $f(u)$ to the nearest integer. Thus, the set consists of $u \in (N, 2N]$ for which $\{f(u)\}$, the fractional part of $f(u)$, lies in $I = [0, \delta) \cup (1-\delta, 1)$. The function f may depend on other parameters than u and, in particular, N. How good an estimate one can make depends on how well-behaved the function f is as well as the size of δ. Certain additional restrictions on f and δ are used to help obtain the estimates. An important aspect of the estimates obtained here are that they are arrived at through the use of finite differences rather than exponential sum techniques.

In addition, the paper shows how such estimates can be used to obtain gap results for $k-$free numbers (integers n such that if p is a prime, then p^k does not divide n), for the distribution of squarefull numbers (integers n such that if p is a prime dividing n, then p^2 divides n), and for the number of non-isomorphic abelian groups of a given order.