A GENERALIZATION OF AN IRREDUCIBILITY THEOREM OF I. SCHUR

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This paper establishes the following generalization of a classical irreducibility result of I. Schur:

Theorem. *Let $a_0, a_1, \ldots, a_n$ denote arbitrary integers with $|a_0| = 1$, and let*

$$f(x) = a_n \frac{x^n}{n!} + a_{n-1} \frac{x^{n-1}}{(n-1)!} + \cdots + a_2 \frac{x^2}{2!} + a_1 x + a_0.$$

If $0 < |a_n| < n$, then $f(x)$ is irreducible over the rationals unless

$$a_n = \pm 5 \quad \text{and} \quad n = 6$$

or

$$a_n = \pm 7 \quad \text{and} \quad n = 10$$

in which cases either $f(x)$ is irreducible or $f(x)$ is the product of two irreducible polynomials of equal degree. If $|a_n| = n$, then either $f(x)$ is irreducible or $f(x)$ is $x \pm 1$ times an irreducible polynomial of degree $n - 1$.

Schur's theorem dealt with the case $a_n = \pm 1$ in the result above. Other related results are discussed. For example, if $|a_n|$ is a prime, then $f(x)$ is also irreducible if $|a_n| > n$.

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