Last Lecture

of

Math 788F

Final Exam:

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Will I be Around? Can be (maybe will be). Please send me email if you would like to get together (or if you have questions).

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Laguerre Polynomials

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Really? What are they?

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Really? What are they?

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THEOREM 7.7.2. $\forall \alpha \in \mathbb{Q} - \mathbb{Z}^-$, $\exists$ finitely many $m \in \mathbb{Z}^+$ such that $L_m^{(\alpha)}(x)$ is reducible.

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We want a prime p that satisfies certain conditions with $g(x)$. One of them is that p does not divide the leading coefficient of $g(x)$. This is clear.

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- Show slope of right-most edge of N. P. of $g(x)$ is $< \frac{1}{k}$.

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Lemma 7.7.7. If $a, b, c, d \in \mathbb{Z}$ with $ad - bc \neq 0$, then the largest prime factor of $(am + b)(cm + d)$ tends to infinity with m.

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Basic Idea in Case $k = 1$:

- Use that if $p|m$ and p is large, then $p|\binom{m}{j}$ for small j and the numerator above for large j.