MATH 785: TRANSCENDENTAL NUMBER THEORY

COURSE DESCRIPTION:

This will be a self-contained course centered around results related to irrational and transcendental numbers. The course will begin by discussing whether certain numbers such as π, e, and e^π are transcendental. We will then turn to general results such as:

(1) The number α^β is transcendental whenever α and β are algebraic numbers with $\alpha \neq 0$, $\alpha \neq 1$, and $\beta \notin \mathbb{Q}$. (Hilbert's seventh problem)

(2) The numbers $e^{\alpha_1}, e^{\alpha_2}, \ldots, e^{\alpha_n}$ are linearly independent over the field of algebraic numbers for any n distinct algebraic numbers $\alpha_1, \ldots, \alpha_n$.

Another topic to be considered involves the expression

$$\Lambda = \beta_0 + \beta_1 \log \alpha_1 + \cdots + \beta_r \log \alpha_r,$$

where $\alpha_1, \ldots, \alpha_r$ and $\beta_0, \beta_1, \ldots, \beta_r$ are non-zero algebraic numbers. The problem here is to show that $|\Lambda|$ is bounded away from 0 (is "big" in some sense) provided certain fairly general conditions hold. All of these are part of transcendental number theory, and in addition to discussing them, we will be interested in two other aspects of the subject: (i) what is *not* known in the subject (for example, is π^e transcendental?) and (ii) how the subject can be applied to other problems (such as estimating the size of the largest prime factor of $2^n - 1$).