MATH 574: TEST 2

Name _____

Instructions: Put your name in the space provided above. Make sure that your test has six different pages including one blank page. Show all your work; the work should be sufficient for me to determine how you derived your answers. Calculators are <u>NOT</u> permitted on this test.

Point Values: Problems (1), (2), (3), (4), and (5) are each worth 12 points, Problem (6) is worth 26 points, and Problem (7) is worth 14 points.

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(1) Write all the 2-combinations of the set $\{a, b, c, d\}$.

(2) A race involves 10 runners. First, second, and third place awards are made. How many possible outcomes are there for the awards? Simplify your answer.

Answer:

(3) How many positive integers ≤ 1000 are divisible by either 2 or 3 (possibly both)? Simplify your answer.

Answer:

(4) A certain department at the University of South Carolina consists of exactly six women and six men. A committee of five people is to be formed having three of the women and two of the men on it. How many committees are possible? Simplify your answer.

Answer:

(5) How many positive integers are there consisting of exactly ten digits all of which are distinct (i.e., each digit from $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is used exactly once in each positive integer)? (Note: 0 is **NOT** to be used for the leading digit.)

Answer:

(6) (a) State the Binomial Theorem.

(b) Calculate $\displaystyle\sum_{k=0}^{n} \binom{n}{k}$ in closed form. Explain your answer.

(c) Calculate $\displaystyle\sum_{k=0}^{n} \binom{n}{k}(-1)^k$ in closed form. Explain your answer.

(d) What is the value of

$$\binom{100}{0} + \binom{100}{2} + \binom{100}{4} + \binom{100}{6} + \cdots + \binom{100}{100}?$$

In other words, what is $\displaystyle\sum_{j=0}^{50} \binom{100}{2j}$? Your answer should be expressed in the form $a \times b^c$ where a, b, and c are explicitly stated numbers (one of them is 1). Justify your answer.

Answer:

(7) Prove that $\displaystyle\sum_{k=0}^{n} \frac{\binom{n}{k}}{k+1} = \frac{2^{n+1}-1}{n+1}$. This is a problem we did in class. You should should show all your work here and not refer to what was done in class.