# Math 574: Quiz 2

Name $\underline{\quad\quad\text{Solution}\quad\quad}$

(1) Let

$$a_1 = \sqrt{90}, \quad a_2 = \sqrt{90 + a_1} = \sqrt{90 + \sqrt{90}}, \quad a_3 = \sqrt{90 + a_2} = \sqrt{90 + \sqrt{90 + \sqrt{90}}}, \ldots,$$

so that $a_{n+1} = \sqrt{90 + a_n}$ for all integers $n \geq 1$. Give a proof that $a_n \leq 10$ for all integers $n \geq 1$ by filling in the boxes below appropriately. There may be more than one way to do this. Note that the proof should consist of complete English sentences. Also, the comments between brackets [ ] are intended to help you determine what to write down and are not part of the actual proof.

**Proof.** We prove that

(∗) $\boxed{a_n \leq 10}$

for all integers $n \geq 1$ by $\boxed{\text{induction on } n}$. Since

$$\boxed{a_1 = \sqrt{90} \leq \sqrt{100} = 10},$$

we see that (∗) holds for $\boxed{n = 1}$. Next, we make our induction hypothesis. Suppose that $\boxed{(*) \text{ holds for some integer } n = k \geq 1}$.

[Be careful in the wording above; note that $k$ is needed here since it is used below.]

Therefore, $\boxed{a_k \leq 10}$. We want to show that $\boxed{a_{k+1} \leq 10}$. From the inequality $\boxed{a_k \leq 10}$, we obtain

$$a_{k+1} = \boxed{\sqrt{90 + a_k} \leq \sqrt{90 + 10} = \sqrt{100} = 10}.$$

[Don't just put what you want to show here. Justify that it is true in this space.]

Therefore, $\boxed{(*) \text{ holds for } n = k + 1.}$

Hence, (∗) holds for every integer $n \geq 1$ by induction, completing the proof. ■