MATH 532, 736I: MODERN GEOMETRY

Name ______________________________

Test #1

Show All Work
Points: Part I (60 pts), Part II (40 pts)

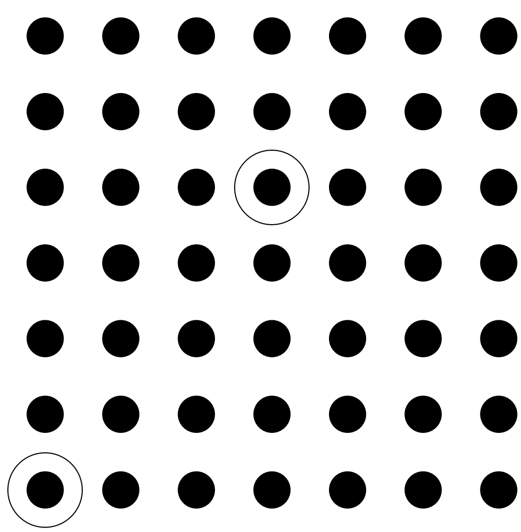
Part I. Each problem in this section is worth 10 points. The last problem, Problem 6, is one of the proofs that you were to have memorized. In Problem 6, I will assume you are using the axioms as you state them in your answer to Problem 2 below.

(1) State the axioms for a finite projective plane of order n.

(2) State the axioms for a finite affine plane of order n. (Number or name the axioms so you can refer to them.)

(3) Give a model for a finite affine plane of order 3. Be sure to clearly mark every point and clearly draw every line in your model.

(4) Two points have been circled in the 7×7 array of points below. Using the model for a finite affine plane of order 7 discussed in class, finish circling the points that belong to the same line as the given circled points.



(5) Consider the points $(2, 8)$ and $(15, 17)$ in a 19×19 array of points for our model of a finite affine plane of order 19. Find the equation of the line passing through these two points. Put your answer in the form $y \equiv mx + k \pmod{19}$ where m and k are among the numbers $0, 1, 2, \ldots, 18$.

(6) Using only the results about affine planes below and the axioms you stated in Problem 2 (and referring to them whenever you use them), prove that in an affine plane of order n, each point has exactly $n+1$ lines passing through it. This is a proof you were to have memorized for class. Note that each result below involves a point and a line. Whenever you use one of the results below, be sure to clarify what point and line you are using with the result.

Result 1: *If ℓ is a line with exactly n points on it and A is a point not on ℓ, then there are exactly $n+1$ lines passing through A.*

Result 2: *If A is a point with exactly $n+1$ lines passing through it and ℓ is a line with A not on ℓ, then there are exactly n points on ℓ.*

(Do NOT prove Result 1 or Result 2!!!)

Part II. The problems in this section all deal with an axiomatic system consisting of the axioms below. Be sure to answer the questions being asked. For example, if you are giving a model to justify your answer in Problem 1 below, make sure you also state whether your answer is, “Yes” or “No.” Problems 1 and 3 and 5 are worth 5 points each, Problem 2 is worth 8 points, and Problem 4 is worth 17 points.

Axiom 1. There exist at least 4 distinct points, no 3 of which are collinear.

Axiom 2. There exists at least 1 line with exactly 2 distinct points on it.

Axiom 3. There exists at least 1 line with at least 3 distinct points on it.

Axiom 4. Given any 2 distinct points, there exists exactly one line passing through the 2 points.

(1) Is the axiomatic system consistent? Justify your answer.

(2) Is the axiomatic system independent? Justify your answer.

(3) Is the axiomatic system complete? Justify your answer.

(4) On this page and the next 3 pages, state the dual of each of the axioms in the indicated space, answer the question whether the dual of the axiom necessarily holds in the axiomatic system, and then provide an explanation for your answer. If you claim that the dual of an axiom holds, then your explanation should consist of an appropriate proof based on the axioms from the axiomatic system. If you claim that the dual of an axiom does not hold, then your explanation can either be to provide an appropriate proof that the dual cannot hold or to provide a model for the axiomatic system which does not satisfy the dual of the axiom. Note that you should give 4 explanations, one for each dual of an axiom.

Dual of Axiom 1:

Does the dual necessarily hold?

Explanation:

Dual of Axiom 2:

Does the dual necessarily hold?

Explanation:

Dual of Axiom 3:

Does the dual necessarily hold?

Explanation:

Dual of Axiom 4:

Does the dual necessarily hold?

Explanation:

(5) Does the principle of duality hold for the axiomatic system? Your answer should be consistent with your answers in the previous problem. To clarify, I will take off for a correct answer if it is inconsistent with the answers in (4), and I will NOT take off for an incorrect answer if it is consistent with the answers in (4). Simply a “Yes” or “No” answer will be sufficient for this problem (but only one will help your test score).