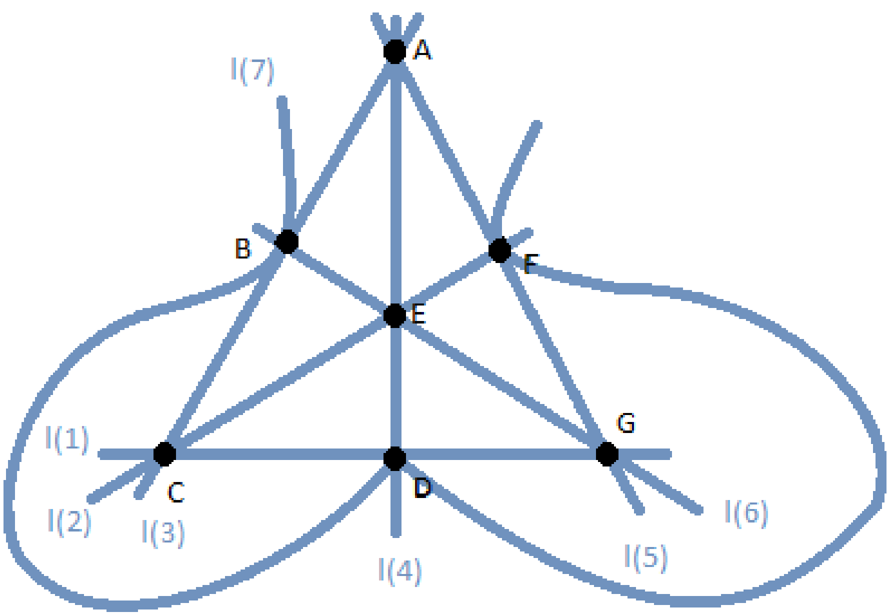
Solutions for Part I

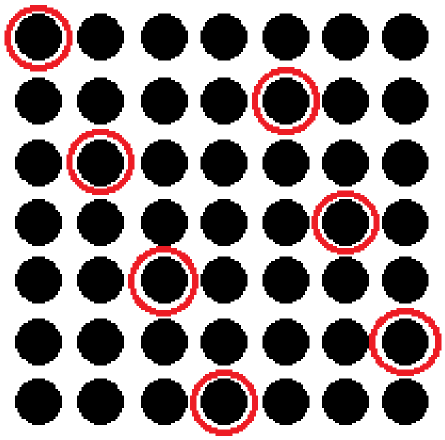
(1) Axiom P1: There exist at least four points, no three of which are collinear.
Axiom P2: There is at least one line that passes through exactly $n+1$ points.
Axiom P3: For any two points, there is exactly one line that passes through both.
Axiom P4: For any two lines, there is at least one point which lies on both of them.

(2) Axiom A1: There exist at least four points, no three of which are collinear.
Axiom A2: There is at least one line that passes through exactly n points.
Axiom A3: For any two points, there is exactly one line passing through them.
Axiom A4: For any line l and any point P not on l there exists exactly one line passing through P that does not intersect l.

(3)



(4) One way to solve this is to see that the two given points have a slope of 4/5 and applying it for each resulting point. Alternatively, set $m = 4/5$ (mod 7) so that $5m = 4$ (mod 7) which gives $m = 5$ (mod 7).Using this at each point gives:



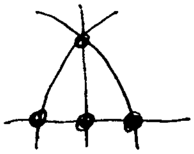
(5) $m = \frac{y_2 - y_1}{x_2 - x_1}$, so $m = \frac{18-1}{16-6} = \frac{17}{10}$. Now we can see that $m = \frac{17}{10} \pmod{19}$ giving us $10m = 17 \pmod{19}$. We use this to find that $m = 15 \pmod{19}$. By using one of the points with $y = mx + b$ we get $1 = 15(6) + b \pmod{19}$ which gives us that $b = 6 \pmod{19}$ and finally $y = 15x + 6 \pmod{19}$.

(6)**Proof.** Let $P_1, P_2, ..., P_{n+1}$ be points on l. Since A is not on l and each P_j is on l, we have that $A \neq P_j$ for each $j \in \{1, 2, ..., n+1\}$. By $\boxed{\text{Axiom P3}}$ there is a line l_j passing through A and P_j for each $j \in \{1, 2, ..., n+1\}$. Since $\boxed{A \text{ is on } l_j}$ and A is not on l, we see that $l_j \neq l$ for each $j \in \{1, 2, ..., n+1\}$. We justify next that the $n+1$ lines $l_1, l_2, ..., l_{n+1}$ are different. Assume $\boxed{l_i = l_j}$ for some i and j in $\{1, 2, ..., n+1\}$ with $i \neq j$. Then the two points $\boxed{P_i \text{ and } P_j}$ are both on l_i. Since these two points are distinct, $\boxed{\text{Axiom P3}}$ implies that there is $\boxed{\text{exactly one line}}$ passing through them. Since $\boxed{P_i \text{ and } P_j}$ are two points that are both on l_i and are both on l, we deduce $\boxed{l_i = l}$. This contradicts that $\boxed{l_i \neq l}$ for any $i \in 1, 2, ..., n+1$. Thus, our assumption is wrong and the lines $l_1, l_2, ..., l_{n+1}$ are different. This finishes the proof that there are at least $n+1$ distinct lines passing through A.

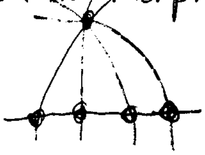
Test 1, Spring 2012

Part II.

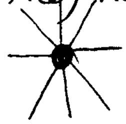
(1) Yes, the axiomatic system is consistent as a model can be drawn that satisfies the system of axioms.



(2)No, since the models are different than the model for (1) we have more than one model, each different (non isomorphic) for the system. Therefore, the axiomatic system is not complete. The models are non-isomorphic since, for example, they have a different number of pts.



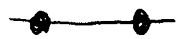
Other models would be



or any finite projective plane would work.

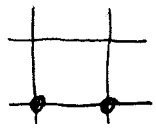
(3)Axiom 1 is independent

∅ , another model would be

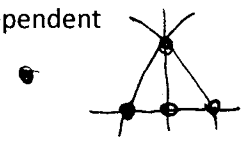


Axiom 2 is independent

, another model would be



Axiom 3 is independent



(4)Dual of Axiom 1:

There exist at least 4 distinct points.

Dual of Axiom 2:

Given any two distinct points, there is at least one line passing through the two points.

Dual of Axiom 3:

Given any two distinct lines, there exists exactly one point they both pass through.

(5) No, Yes, Yes

(6)No, since the dual of Axiom 1 does not hold for all models satisfying the axioms in this axiomatic system. The dual of Axiom 1 does not hold for the second model in (2) above.