MATH 532, 736I: MODERN GEOMETRY

Test 1, Spring 2012 Name ________________________________

Show All Work

Instructions: This test consists of 3 pages of problems. Put your name at the top of this page and at the top of the first page of the packet of blank paper given to you. Work each problem in the packet of paper unless it is indicated that you can or are to do the work below. Show <u>ALL</u> of your work. Do <u>NOT</u> use a calculator.

Points: Part I (56 pts), Part II (44 pts)

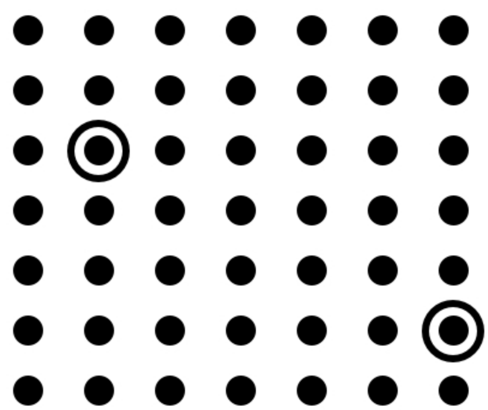
Part I. The point value for each problem appears to the left of each problem. In Problem 6, I will assume you are using the axioms as you state them in your answer to Problem 1 below.

8 pts (1) In the packet of white paper provided to you, state the axioms for a finite PROJECTIVE plane of order n. (Number or name the axioms so you can refer to them in Problem 6.)

8 pts (2) In the packet of white paper provided to you, state the axioms for a finite AFFINE plane of order n.

8 pts (3) Draw a model for a finite PROJECTIVE plane of order 2. Be sure to clearly mark every point and clearly draw every line in your model. If I look at your model, I should be able to tell where each of your points and lines are. In particular, make sure that each line you draw cannot be mistaken for two lines.

8 pts (4) Two points have been circled in the 7×7 array of points to the right. Using the model for a finite affine plane of order 7 discussed in class, finish circling the points that belong to the same line as the given circled points. (You do not need to use the packet of blank paper for this problem.)



8 pts (5) Consider the points $(6, 1)$ and $(16, 18)$ in an 19×19 array of points for our model of a finite affine plane of order 19. Find the equation of the line passing through these two points. Put your answer below in the form $y \equiv mx + k \pmod{19}$ where m and k are among the numbers $0, 1, 2, \ldots, 18$. Be sure to show your work in the packet provided with this test. You will not get credit for a correct answer without correct work.

Answer: [ ]

16 pts

(6) Using the axioms you stated in Problem 1, fill in the boxes below to complete a proof of the result stated below. There are two "double" boxes (a box inside of a box) below. These two double boxes should be completed in the same way (that is, whatever you put in one double box should be the same as what you put in the second double box).

Result: *If ℓ is a line with exactly $n+1$ points on it in a finite projective plane of order n and A is a point not on ℓ, then there exist at least $n+1$ distinct lines passing through A.*

This is part of a result done in class and that you were to have learned for this test.

Proof. Let $P_1, P_2, \ldots, P_{n+1}$ be the $n+1$ points on ℓ. Since A is not on ℓ and each P_j is on ℓ, we have that $A \neq P_j$ for each $j \in \{1, 2, \ldots, n+1\}$. By ________, there is a line ℓ_j passing through A and P_j for each $j \in \{1, 2, \ldots, n+1\}$. Since ________ and A is not on ℓ, we see that $\ell_j \neq \ell$ for each $j \in \{1, 2, \ldots, n+1\}$. We justify next that the $n+1$ lines $\ell_1, \ell_2, \ldots, \ell_{n+1}$ are different. Assume ________ for some i and j in $\{1, 2, \ldots, n+1\}$ with $i \neq j$. Then the two points ________ are both on ℓ_i. Since these two points are distinct, ________ implies that there is ________ passing through them. Since ________ are two points that are both on ℓ_i and are both on ℓ, we deduce ________. This contradicts that ________. Thus, our assumption is wrong and the lines $\ell_1, \ell_2, \ldots, \ell_{n+1}$ are different. This finishes the proof that there are at least $n+1$ distinct lines passing through A. ■

Part II. The point values appear to the left of each problem. The problems in this section all deal with an axiomatic system consisting of the following axioms.

Axiom 1. There exist at least 4 distinct lines.

Axiom 2. Given any two distinct lines, there is at least one point on both of them.

Axiom 3. Given any two distinct points, there exists exactly one line passing through them.

[8 pts] (1) Justify that the axiomatic system is consistent.

[8 pts] (2) Justify that the axiomatic system is *not* complete. Include some brief explanation, in complete English sentences, for your answer.

[12 pts] (3) Justify that the axiomatic system is independent.

[6 pts] (4) State the duals of each axiom. You should have one dual for each axiom.

[6 pts] (5) For each dual in Problem 4, decide whether the dual is true in the axiomatic system - that is, does it hold for all models satisfying the axioms in the axiomatic system (not just the models you found, but for all models)? After deciding whether the dual of each axiom holds for every model satisfying the axioms in the axiomatic system, answer the questions below with either "Yes" or "No". You do not need to give an explanation for your answers on this problem.

Does the dual of Axiom 1 hold for all models for the axiomatic system? [ ]

Does the dual of Axiom 2 hold for all models for the axiomatic system? [ ]

Does the dual of Axiom 3 hold for all models for the axiomatic system? [ ]

[4 pts] (6) Does the principle of duality hold for this axiomatic system? Answer the question below, and then give a very brief explanation for you answer. This answer should be consistent with the answers you gave in Problem 5 above.

Does the principle of duality hold? [ ]

Explanation: [ ]