Math 532: Quiz 3

Name KEY

Axiom 1. There exist exactly 3 points.
Axiom 2. Given any 2 distinct points, there exists exactly one line passing through the 2 points.
Axiom 3. Given any line, there is a point not on the line.
Axiom 4. Any two lines intersect in at least one point.

1. Finish the proof below that, for an axiomatic system with the axioms above, each line has exactly two points on it.

Proof: Let ℓ be a line. By Axiom 3, there is a point $\boxed{\text{not on } \ell}$.

By Axiom 1, there are exactly 3 points. Therefore, ℓ has $\boxed{\leq}$ 2 points on it. Assume
$\leq$ or $\geq$

ℓ has $\boxed{<}$ 2 points on it. Then there are at least $\boxed{2}$ points *not* on ℓ. Let ℓ' be a
$<$ or $>$

line passing through 2 of these points, say A and B. We know ℓ' exists by Axiom $\boxed{2}$.

Observe that $\ell' \neq \ell$ since $\boxed{A \text{ is on } \ell' \text{ but not on } \ell}$.

By Axiom 4, ℓ and ℓ' have at least one point, say P, in common. We know that $P \neq A$ and $P \neq B$ since $\boxed{A \text{ and } B \text{ are not on } \ell \text{ and } P \text{ is on } \ell}$. Since Axiom 1 indicates that there are exactly 3 points, these 3 points must be $\boxed{A, B \text{ and } P}$. This contradicts Axiom $\boxed{3}$ since $\boxed{A, B \text{ and } P \text{ are all on } \ell'}$.

Therefore, ℓ must have exactly 2 points on it, finishing the proof. ■