MATH 532/736I, LECTURE 5

1. Finish Projective Plane Theorems from Homework 2
2. Homework: Mention Modulo Arithmetic Homework
3. Define a modulo m. Discuss and prove how congruences can be treated almost like equations (see theorems below).
4. **Theorem 1.** Let a, b, c, d, and m be integers with $m > 0$. Then the following are true:
   - If $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$, then $a \equiv c \pmod{m}$.
   - If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $a \pm c \equiv b \pm d \pmod{m}$.
   - If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $ac \equiv bd \pmod{m}$.
   - If $a \equiv b \pmod{m}$, then $a^k \equiv b^k \pmod{m}$ for every $k \in \{0, 1, 2, \ldots\}$.
5. **Theorem 2.** Let a and m be positive integers with no common prime divisors. Then there is a positive integer b such that $ab \equiv 1 \pmod{m}$.
6. **Examples:**
   - Is 25621235904 divisible by 3? . . . by 4? . . . by 5? . . . by 9? . . . by 11?
   - What is the last digit of 347^{223}?
   - Is 5638287462039703 the sum of 2 squares?
   - A Fermat number is a number of the form $F_n = 2^{2^n} + 1$. The first few Fermat numbers are

     $$F_0 = 3,\ F_1 = 5,\ F_2 = 17,\ F_3 = 257,\ F_4 = 65537, \text{and} \quad F_5 = 4294967297.$$

     The first 5 Fermat numbers are primes. Explain why F_5 is divisible by 641 using that $641 = 5 \times 2^7 + 1 = 2^4 + 5^4$.
7. **Homework:**
   (1) Test the number 1433456304672354 for divisibility by 2, 3, 4, 5, 9, and 11.
   (2) Make up a test for divisibility by 8 and explain why it works.
   (3) What is the last digit of $1^{170469} + 2^{170469} + 3^{170469} + \cdots + 100^{170469}$?
   (4) What is the last digit of $12^{(34^{56})}$?
   (5) Using an argument modulo 8, explain why 3462437654807 is not the sum of 3 squares?