MATH 532/736I, LECTURE 1

1. Hand out and go over syllabus.

2. Class photos.

3. No homework (today).

4. **Logic in the Last Century:**

   - Are there statements that can be made in mathematics which are true but which we cannot prove?

     Remark 1: The answer cannot be "Yes" since if there exist such statements, we would not know they are true, so we would not know they exist.

     Remark 2: Remark 1 is wrong. Such statements are known to exist (but that does not mean that we know what they are).

   - Is mathematics consistent? Is it possible that some day a proof will exist that $1+1=3$?

     Remark 1: If mathematics were not consistent, we wouldn't have this class. Therefore, mathematics must be consistent.

     Remark 2: Remark 1 is questionable. No proof exists that mathematics is consistent. It is known, however, that if mathematics is consistent, then we cannot prove it is consistent.

5. **Back to Euclid's Time:**

   (i) Constructions:

   - Find the perpendicular bisector of a line segment.
   - Duplicate an angle.
   - Divide a segment into 3 equal pieces.
   - Given a line ℓ and a point P not on ℓ, construct a line ℓ' parallel to ℓ that passes through P.
   - Find the center of a given circle.
   - Given a line $\overleftrightarrow{DE}$ and two points A and B on one side of the line, find C on $\overleftrightarrow{DE}$ so that $\angle ACD = \angle BCE$.
   - Given a circle C and a point P outside C, construct a line ℓ tangent to C that passes through P.
   - Given a circle C and two points P and Q outside C, construct the circles that are tangent to C and pass through P and Q.

(ii) Even More Basic Constructions:

   - Given two points, construct a line through them. (What if the points are far apart and the straightedge and compass are small by comparison?)
   - Draw a circle with a given radius and center.

(iii) Euclid's Axioms (or Postulates):

   1. A straight line can be drawn through any 2 points.
   2. A straight line can be extended in either direction as long as we wish.
   3. Given any point P and any distance r, we can draw a cirlce of radius r centered at P.
   4. All right angles are equal.
   5. If a line ℓ intersects 2 lines ℓ_1 and ℓ_2 and makes 2 interior angles on the same side of ℓ each less then a right angle, then the lines ℓ_1 and ℓ_2 intersect on that side of ℓ.

   Comment: From these axioms, Euclid developed theorems (or propositions).

6. **Between Euclid and a Century Ago:**

   - What if the axioms were different? Consider something like:
     1. There exist 3 non-collinear points.
     2. Given two points, there exists a line passing through them.
     3. Any two distinct lines intersect in exactly two points.

     Do these axioms make sense? Are they simply describing something that's not true and therefore nonsense?
   - Describe great circles on a sphere, and explain why the axioms above all hold in this context.