Math 241: Test 3

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Make sure that your test has eight different pages including one blank page. Work each problem below and show ALL of your work. You do not need to simplify your answers. Do NOT use a calculator.

Point Values: Problem (1) is worth 16 points, and each of the remaining problems (Problems (2) through (7)) is worth 14 points.

(1) Calculate the following double integrals. **SIMPLIFY** your answers.

(a) $\int_0^1 \int_0^{\sqrt{x}} \sqrt{x}\, dy\, dx$

Answer: $\boxed{}$

(1) (continued)

(b) $\displaystyle\int_0^{\pi}\int_0^{2} \theta \, dr \, d\theta$

Answer: $\boxed{}$

(c) $\displaystyle\int_{-1}^{1}\int_{y^2}^{1} \cos(x^{3/2}) \, dx \, dy$ (Your answer should involve a trigonometric function.)

Answer: $\boxed{}$

(2) Write each iterated integral below as an iterated integral with the order of integration interchanged.

(a) $\int_0^1 \int_0^x f(x, y)\, dy\, dx$

Answer: $\boxed{}$

(b) $\int_0^1 \int_{x^3}^1 f(x, y)\, dy\, dx$

Answer: $\boxed{}$

(3) (a) Let $R = \{(x, y) : 0 \leq x \leq 3,\, 0 \leq y \leq 3\}$ and

$$f(x,y) = \begin{cases} 3 & \text{if } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 2 \\ -1 & \text{if } 0 \leq x \leq 2 \text{ and } 2 < y \leq 3 \\ 2 & \text{if } 2 < x \leq 3 \text{ and } 0 \leq y \leq 3. \end{cases}$$

Evaluate $\displaystyle\iint_R f(x, y)\, dA$.

Answer: $\boxed{}$

(b) Calculate $\displaystyle\int_0^3 \int_0^1 f(x, y)\, dy\, dx$ where $f(x, y)$ is as given in part (a).

Answer: $\boxed{}$

(4) Calculate cylindrical coordinates (r, θ, z) and spherical coordinates (ρ, θ, ϕ) for the point with rectangular coordinates $(x, y, z) = (\sqrt{2}, \sqrt{2}, 2)$. Simplify your answers so that no trigonometric functions are used.

(r, θ, z): $\boxed{}$

(ρ, θ, ϕ): $\boxed{}$

(5) Express the volume of the solid in the first octant bounded by the coordinate planes, the surface $y^2 + z^2 = 4$ and the plane $x = 3$ as an iterated triple integral. Do not evaluate the integral.

Answer: $\boxed{}$

(6) Express the volume of the solid above the surface $z = x^2 + y^2$ and below the surface $x^2 + y^2 + z^2 = 6$ as an iterated integral in polar or cylindrical coordinates. Do not evaluate the integral. (Hint: If (x, y, z) is a point on both surfaces, then $z + z^2 = 6$.)

Answer:

(7) Using spherical coordinates, calculate

$$\iiint_S \left(x^2 + y^2 + z^2\right)^{3/2} \, dV$$

where S is the solid bounded above by the sphere $x^2 + y^2 + z^2 = 1$ and below by the plane $z = 0$. Simplify your answer.

Answer: $\boxed{}$