MATH 241: FINAL EXAM

Name _____

Instructions and Point Values: Put your name in the space provided above. Check that your test contains 11 different pages including one blank page. Work each problem below and show <u>ALL</u> of your work. Unless stated otherwise, you do not need to simplify your answers. Do <u>NOT</u> use a calculator.

There are 100 total points possible on this exam. The points for each problem in each part is indicated below.

PART I:

Problem (1) is worth 8 points.
Problem (2) is worth 6 points.
Problem (3) is worth 6 points.
Problem (4) is worth 7 points.
Problem (5) is worth 8 points.
Problem (6) is worth 9 points.
Problem (7) is worth 8 points.

PART II:

Problem (1) is worth 12 points.Problem (2) is worth 12 points.Problem (3) is worth 12 points.Problem (4) is worth 12 points.

PART I.

- (1) Let $\vec{u} = \langle 2, 1, -2 \rangle$ and $\vec{v} = \langle 4, -1, -1 \rangle$. Calculate
- (a) $2\overrightarrow{u} \overrightarrow{v}$



(b) $|\vec{u}|$ (the magnitude of \vec{u})

Answer:

(c) the angle between \vec{u} and \vec{v} (simplify so no inverse trig functions are in your answer)

Answer:

(d) $\vec{u} \times \vec{v}$

Answer:	
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(2) Calculate an equation for the tangent plane to the surface

$$xy + xz + yz = 3$$

at the point (1, 1, 1).

Answer:

- (3) For both parts of this problem, $f(x, y) = x^2y y^3 + 2y$.
- (a) Find the directional derivative of f(x, y) at the point (2, -1) in the direction of $\langle 1, 2 \rangle$.

Directional Derivative:



(b) There are infinitely many different values for the directional derivative of f(x, y) at the point (2, -1). Find a unit vector \vec{u} such that the directional derivative of f(x, y) in the direction of \vec{u} is maximal (as large as possible).

(4) Let

$$f(x,y) = \lim_{h \to 0} \frac{(x+2y+h)^{3/2} - (x+2y)^{3/2}}{h}.$$

Calculate f(2, 1). (Comment: If you get the answer 6, then you probably have the basic idea right but not the answer.)

Answer:

(5) (a) Calculate the line integral

$$\int_{\mathcal{C}} (x^2 \cos y + y) \, dx + (x^2 \cos y - y) \, dy$$

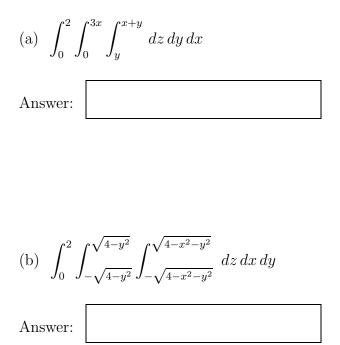
where C is the line segment from (1,0) to (0,1).

Answer:	
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(b) What is the value of the line integral in part (a) if instead C is the line segment from (0, 1) to (1, 0)?

Answer:

(6) Calculate each of the following integrals.



(c) $\int_0^1 \int_x^{\sqrt{2-x^2}} (x^2 + y^2)^{3/2} \, dy \, dx$

Answer:

(7) Let

$$f(x,y) = xy(x+2) e^{-2y^2}.$$

Then

$$f_x = 2y(x+1) e^{-2y^2}, \qquad f_y = -x(x+2)(2y-1)(2y+1) e^{-2y^2}$$

 $f_{xx} = 2y e^{-2y^2}, \qquad f_{yy} = 4xy(x+2)(4y^2-3) e^{-2y^2},$

and

$$f_{xy} = -2(x+1)(2y-1)(2y+1)e^{-2y^2}.$$

The function f(x, y) has four critical points. Calculate the four critical points and indicate (with justification) whether each determines a local maximum value of f(x, y), a local minimum value of f(x, y), or a saddle point of f(x, y).

	Critical Point	Local Min., Local Max., or Saddle Pt.
First		
Second		
Third		
Fourth		

PART II.

(1) Let \mathcal{P} be the plane given by the equation x - y + z = 2. The point Q = (-1, 2, 1) is not on the plane \mathcal{P} . Find the equation of a plane which passes through the point Q and is *perpendicular* to the plane \mathcal{P} .

Answer:

(2) Using Green's Theorem, calculate the line integral

$$\int_{\mathcal{C}} \left(\cos x + \sin y - xy^{3}\right) dx + \left(x\cos y - x^{2}y^{2} + e^{y^{2}+1}\right) dy$$

where C is the rectangle oriented counter-clockwise with vertices (0,0), (1,0), (1,3), and (0,3). Simplify your answer.

Answer:	
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(3) Determine the maximum and the minimum values of the function

$$18x^2 - 6x + 3 - 24xy + 16y^2$$

on the triangle $\{(x, y) : 0 \le x \le 1, 0 \le y \le x\}$. Simplify your answers.

Maximum Value:	
Minimum Value:	

(4) Calculate the volume of the solid lying above the xy-plane and inside the surfaces

$$z^2 = -1 + x^2 + y^2$$
 and $3x^2 + 3y^2 + z^2 = 4$.

Simplify your answer. (Hint: Express the volume as a sum of two integrals.)

Answer:	
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