Green's Theorem - Quick Version

Green's Theorem - Very Quick

Definition: Let $\mathcal{C}$ be a curve in the xy-plane (say $x = x(t)$ and $y = y(t)$ with $a \leq t \leq b$). Then the *line integral*, denoted

$$\int_{\mathcal{C}} f(x, y)\, ds,$$

is the area of the region directly above this curve and below the surface $z = f(x, y)$.

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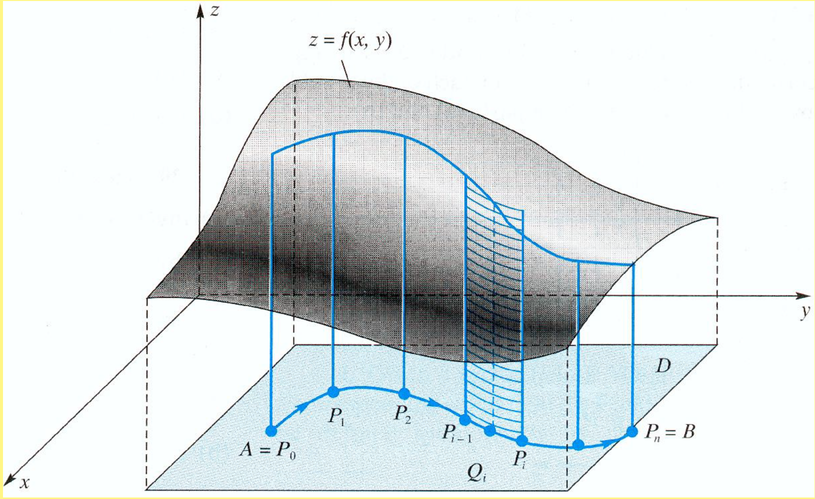

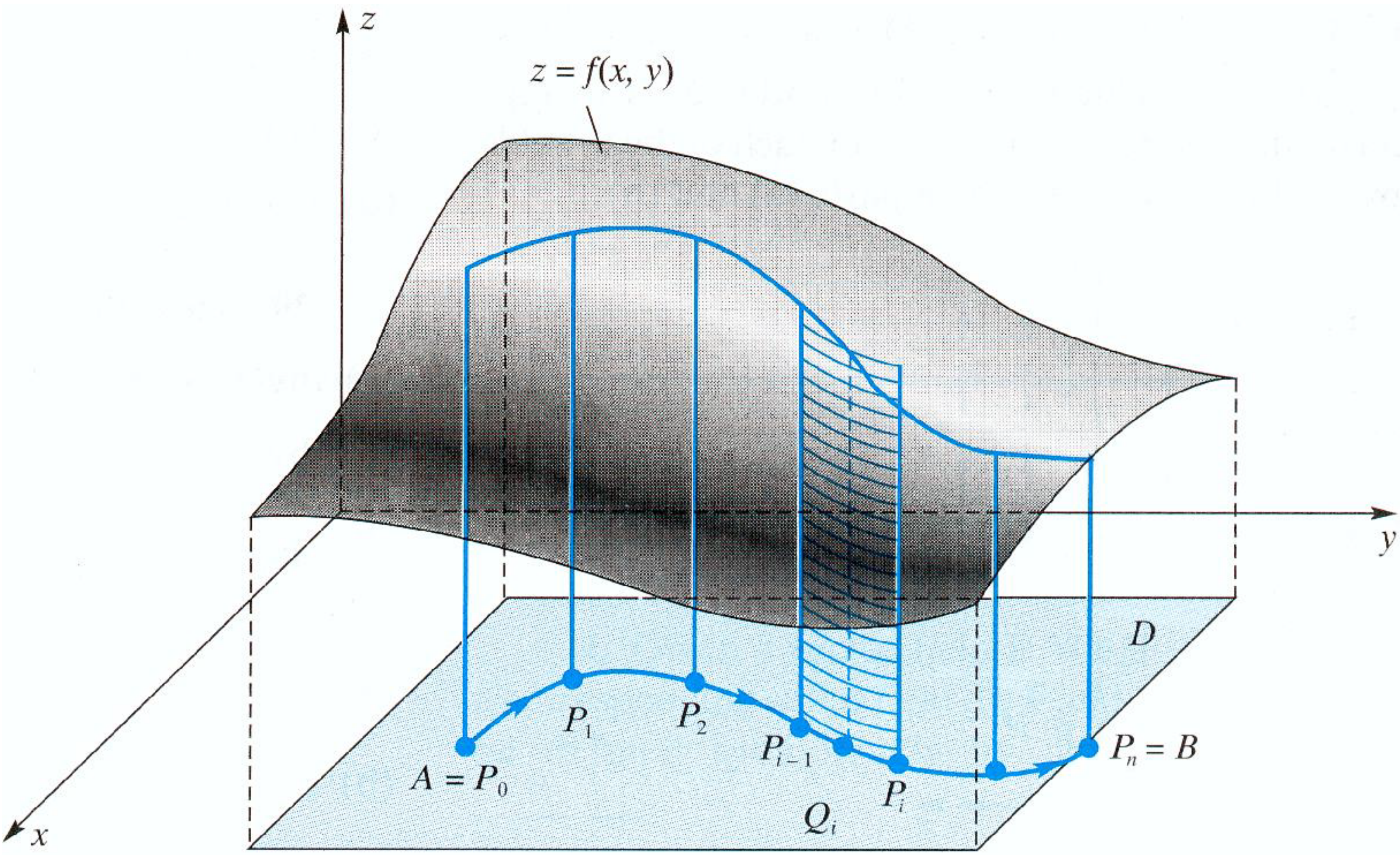
z
z = f(x, y)
y
x
D
A = P_0
P_1
P_2
P_{i-1}
P_i
Q_i
P_n = B

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$$= \int_a^b \left(M(x,y)\,\frac{dx}{dt} \;+\; N(x,y)\,\frac{dy}{dt} \right) dt.$$

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Green's Theorem: Let $\mathcal{C}$ be a piecewise smooth, simple closed curve having a counterclockwise orientation that forms the boundary of a region S in the xy-plane. If $M(x, y)$ and $N(x, y)$ have continuous partial derivatives on S and its boundary $\mathcal{C}$, then

$$\oint_{\mathcal{C}} M(x, y)\, dx + N(x, y)\, dy = \iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA.$$