Math 241: Quiz 8

Show ALL Work Name **Solutions**

1. The function
$$f(x, y) = 5 - x^3 + 27x + y^2 - 4y$$
has two critical points. Find the points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ where the critical points occur, and determine whether there is a local maximum, a local minimum or a saddle point at each of the points.

$P_1 =$ $(3, 2)$ (indicate point) is a location of a **saddle point** (local max, local min, or saddle pt)

$P_2 =$ $(-3, 2)$ (indicate point) is a location of a **local minimum** (local max, local min, or saddle pt)

Solution: Setting $f_x = 0$ and $f_y = 0$, we get $-3x^2 + 27 = 0$ and $2y - 4 = 0$. The first of these gives $x = \pm 3$ and the second gives $y = 2$. So $P_1 = (3, 2)$ and $P_2 = (-3, 2)$ (or the other way around). Since $D = f_{xx}f_{yy} - f_{xy}^2 = (-6x) \cdot 2 = -12x$, we see that $D(3, 2) = -36 < 0$, so $(3, 2)$ is the location of a saddle point. Also, $D(-3, 2) = 36 > 0$. Since $f_{xx}(-3, 2) = 18 > 0$, there is a local minimum point at $(-3, 2)$. ■

2. Calculate the minimum distance from the point $(1, 2, 1)$ to a point on the hyperboloid of one sheet given by
$$\frac{x^2}{2} - y^2 + (z - 1)^2 = 1.$$

Minimum Distance: $\sqrt{2}$

Solution: We want to minimize the distance from a point (x, y, z) on the hyperboloid to the point $(1, 2, 1)$. We minimize the distance squared instead which is $(x-1)^2+(y-2)^2+(z-1)^2$. Since the point (x, y, z) is on the hyperboloid, we can use that $(z - 1)^2 = 1 - (x^2/2) + y^2$ (from the equation of the hyperboloid). So the function we want to minimize is $f(x, y) = (x-1)^2+(y-2)^2+1-(x^2/2)+y^2$. From $f_x = 2(x-1)-x = 0$ and $f_y = 2(y-2)+2y = 0$, we obtain $x = 2$ and $y = 1$. Since $(x, y, z) = (2, 1, z)$ is on the hyperboloid, we also have $(2^2/2) - 1^2 + (z - 1)^2 = 1$. This gives $z = 1$. So the point on the hyperboloid that gives the minimum (and note a minimum must exist) is $(2, 1, 1)$. We deduce that the minimum distance is the distance from $(2, 1, 1)$ to the point $(1, 2, 1)$, which is $\sqrt{2}$. ■