Math 241: Quiz 8

Show ALL Work Name **Solutions**

1. Determine the absolute maximum and absolute minimum value of

$$f(x,y) = 3x^2 + 2y^2 - 3x - 4$$

where (x,y) varies over the points satisfying $x^2+y^2 \leq 4$. Furthermore, indicate ALL points (x,y) satisfying $x^2+y^2 \leq 4$ where these values occur.

Absolute Maximum Value: $\boxed{14}$ ⟵ **This value occurs at:** $\boxed{(-2,0)}$

Absolute Minimum Value: $\boxed{-19/4}$ ⟵ **This value occurs at:** $\boxed{(1/2,0)}$

Solution. For (x,y) satisfying $x^2+y^2<4$, we want

$$f_x = 6x - 3 = 0 \quad \text{AND} \quad f_y = 4y = 0.$$

So we have only one point $(1/2,0)$ to consider here. We note that $f(1/2,0) = -19/4$. For (x,y) satisfying $x^2+y^2=4$, we have $y^2 = 4-x^2$ and $-2 \leq x \leq 2$, so $f(x,y) = g(x)$ where

$$g(x) = 3x^2 + 2(4-x^2) - 3x - 4 = x^2 - 3x + 4.$$

So we want to maximize and minimize $g(x)$ where $-2 \leq x \leq 2$ (a first year Calculus problem). Since $g'(x) = 2x - 3 = 0$ precisely when $x = 3/2$ and $3/2 \in [-2,2]$, we only need to consider the points where $x = -2, 3/2$ and 2 (note endpoints must be considered). Since $g(-2) = 14$, $g(2) = 2$ and $g(3/2) = 7/4$, the maximum value of $f(x,y)$ on the circle $x^2+y^2=4$ is 14 and the minimum is $7/4$. For $x=-2$ and $x^2+y^2=4$, we have $y=0$; and for $x=3/2$ and $x^2+y^2=4$, we have $y = \pm\sqrt{7}/2$. Recalling $f(1/2,0) = -19/4$, we get the answers above and the answers below (on the next page). For those of you who are more organized, you might want to make a table similar to the following as you go along. ■

Location	x	y	Crit. Pts.	Value of f	Conclusion
inside	$1/2$	0	$(1/2,0)$	$-19/4$	abs. min.
boundary	$3/2$	$\pm\sqrt{7}/2$	$(3/2, \pm\sqrt{7}/2)$	$7/4$	abs. min. on boundary
boundary	-2	0	$(-2,0)$	14	abs. max. (on boundary)
boundary	2	0	$(2,0)$	2	nothing worth noting

2. For this problem, you should only have to use the work you already did above. Determine the absolute maximum value and the absolute minimum value of

$$f(x, y) = 3x^2 + 2y^2 - 3x - 4$$

where (x, y) varies over the points satisfying $x^2 + y^2 = 4$. Furthermore, indicate ALL points (x, y) satisfying $x^2 + y^2 = 4$ where these values occur.

Absolute Maximum Value: $\boxed{14}$ ⟵ **This value occurs at:** $\boxed{(-2, 0)}$

Absolute Minimum Value: $\boxed{7/4}$ ⟵ **This value occurs at:** $\boxed{(3/2, \pm\sqrt{7}/2)}$