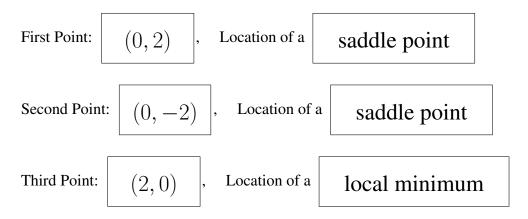
Math 241: Quiz 7

Show ALL Work

Name

Solutions

1. Determine all points (x, y) where $f(x, y) = x^2 - 4x + xy^2$ has a local minimum, a local maximum or a saddle point. There should be 3 such points. Indicate next to each point, whether the point is the location of a local minimum, a local maximum or a saddle point.



Solution: Setting the partial derivatives to 0, we obtain

$$f_x = 2x - 4 + y^2 = 0$$
 AND $f_y = 2xy = 0.$

The second of these implies that x = 0 OR y = 0. If x = 0, then the first equation gives $-4 + y^2 = 0$, so $y = \pm 2$. If y = 0, then 2x - 4 = 0, so x = 2. This gives us the three points (0, 2), (0, -2) and (2, 0). For "local" extrema, we want to use

$$D(x,y) = f_{xx}f_{yy} - f_{xy}^2 = 2 \cdot 2x - (2y)^2 = 4x - 4y^2.$$

Since $D(0, \pm 2) = -16 < 0$, there are saddle points at (0, 2) and (0, -2). Since D(2, 0) = 8 > 0 and $f_{xx}(2, 0) = 2 > 0$, there is a local minimum at (2, 0).