MATH 141: FINAL EXAM

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Check that your test contains 14 different pages including one blank page. Work each problem below and show ALL of your work. Unless stated otherwise, you do not need to simplify your answers. You should NOT use Calculus material in your answers that are unrelated to this course. Do NOT use a calculator.

There are 300 total points possible on this exam. There are 2 parts. The first part consists of 20 problems each worth 10 points. The second part consists of 5 problems each worth 20 points.

PART I. Calculate each of the following. An asterisk ($*$) next to a problem number indicates that you do not need to show work for that problem. You should show work for every other problem.

(1) The equation of the tangent line to the graph of $y = x^2$ at the point $(1, 1)$.

(2)* $\displaystyle\lim_{t\to 2} \frac{t^2 - 4}{t^2 + 4}$

(3)* $\lim_{x \to 0^-} \frac{|x|}{x}$

(4) $\lim_{x \to 1^+} \frac{x^2 - 1}{\sqrt{x - 1}}$

(5) $\lim_{x \to 0} \frac{x^2}{(\cos x) - 1}$

(6) $\lim_{h \to 0} \dfrac{\sin(\pi + h) - \sin \pi}{h}$

(7) $\lim_{x \to \infty} \left(\sqrt{x^2 + 3x} - \sqrt{x^2 - 3x} \right)$

(8)* $\dfrac{d}{dx} \left((x+1)^{20} \sqrt{x} \right)$

(9)* $\dfrac{d}{dx}\left(\dfrac{\sqrt{x}}{(x+1)^{20}}\right)$

(10)* $f'(t)$ if $f(t) = \sin(3t+1)$

(11) $y = f(x)$ if $xy' + xy^2 = \dfrac{y^2}{x}$ and if $f(1) = 1$

(12) $\dfrac{d^5}{dx^5}\left(2x^4 + 33x^3 - 23x + 7\right)$

(13) $\dfrac{d}{dx}\left(f(1/x)\right)$ if $f'(x) = 10^x$

(14) $\displaystyle\int_0^{\pi/4} \sin(4t)\, dt$

(15) $\displaystyle\int \frac{\cos\sqrt{t}}{\sqrt{t}}\,dt$

(16) $\displaystyle\int x\sqrt{x+2}\,dx$

(17) $\displaystyle\int \frac{(x+1)^2}{\sqrt{x}}\,dx$

(18) $\dfrac{d}{dx}\left(\int_x^{x^2} \cos\sqrt{t}\,dt\right)$

(19) $\displaystyle\int_0^{\pi^2} \left(\frac{d}{dt}(\cos\sqrt{t})\right)\,dt$

(20) The area of the region bounded by the graphs of $x = y^2$ and $x = 1$.

PART II. Answer each of the following. Make sure your work is clear. If you do not know how to answer a problem, tell me what you know that you think is relevant to the problem. If you end up with an answer that you think is incorrect, tell me this as well. Better yet, tell me why you think it is incorrect. In other words, let me know what you know.

(1) The point $(1, 1)$ is on the graph of $x^2y + xy^3 = 2$. Calculate the equation of the tangent line to the graph of $x^2y + xy^3 = 2$ at the point $(1, 1)$.

(2) Each of the following refers to the function $f(x) = 3x^5 - 10x^3$.

(a) Find all the critical points for $y = f(x)$.

(b) Find all local maximum values of $f(x)$.

(c) Find all local minimum values of $f(x)$.

(d) Find all the inflection points for $y = f(x)$.

(e) On what intervals is the graph of $y = f(x)$ concave up?

(f) Graph $y = f(x)$.

(3) Part (a) below can be done with or without Calculus. If you happen to know a formula for the distance described in (a), you should justify that the formula works before using it. Part (b) below depends on Part (a), but you do not need to do Part (a) correctly to do Part (b) correctly. You should try Part (b) even if you feel you cannot do Part (a).

(a) Let P be a point (a, b), and suppose that P is to the right of the line $y = x$. This means $a > b$. Show that the distance (in other words, the "minimal" distance) from P to the line $y = x$ is $(a - b)/\sqrt{2}$.

(b) Consider the points (a, b) on the graph of $y = x^3$ with $0 \leq a \leq 1$. Determine which of these points (a, b) is the greatest distance from the line $y = x$.

(4) Calculate the volume of the solid obtained by revolving the region above $y = x^2$ and below $y = 1$ about the x-axis. Simplify your answer.

(5) The graph of $y = \dfrac{\sin x}{x}$ intersects the x-axis at $x = 2\pi$ and $x = 3\pi$ (and at some other points too). The graph is above the x-axis for $2\pi < x < 3\pi$. Let R be the region below the graph of $y = \dfrac{\sin x}{x}$ and above the x-axis (you do not need to know exactly what this region looks like). Calculate the volume of the solid obtained by revolving the region R about the y-axis. Simplify your answer.