MATERIAL TO STUDY FOR MATH 780

I have tried to list below the main topics covered in the course. If a topic is not to be included as part of the comprehensive exam, I have written "omit" next to the topic. If I expect you to know the proof(s) associated with a given topic, I have written "proofs" next to the topic.

Part I. The Basics

(1) Irrational Numbers (omit)
(2) The Fundamental Theorem of Arithmetic
(3) Euclidean Algorithm
(4) Continued Fractions (omit)
(5) Modulo Arithmetic (proofs)
(6) Fermat's Little Theorem
(7) Pseudoprimes, probable primes, and absolute pseudoprimes
(8) Euler's Theorem, a generalization of (6) (proofs)
(9) Wilson's Theorem
(10) Public-Key Encryption (omit)
(11) The Chinese Remainder Theorem (proofs)
(12) Euler's Phi Function
(13) Lagrange's Theorem about roots of polynomials modulo a prime
(14) Primitive Roots
(15) Euler's Criterion for when a number is a square modulo a prime
(16) Calculating Legendre symbols
(17) Quadratic Reciprocity
(18) Solving polynomial equations modulo a composite number
(19) When is n a sum of 2 squares? (omit)
(20) Tossing coins over the phone (omit)

Part II. Other Material

(1) Asymptotic notation
(2) The sum of the reciprocals of the primes diverges (proofs)
(3) Products and other sums involving primes
(4) The normal number of prime divisors of n
(5) Chebyshev's Theorem
(6) The Prime Number Theorem and its generalizations
(7) Riemann-Stieltjes' Integrals and their connection to (2) and (3)
(8) The density of integers with large prime factors
(9) The density of primes
(10) The density of squarefree numbers (proofs)
(11) Fermat's Last Theorem for almost all exponents (omit)
(12) The Pure Brun Sieve (omit)
(13) The sum of the reciprocals of the twin primes converges (omit)
(14) Irreducibility Results (omit)