PAlmetto Number Theory Series

University of South Carolina, December 3-4, 2016

Plenary Speakers

Nigel Boston (University of Wisconsin at Madison)

Title: *Non-abelian Cohen-Lenstra heuristics*

Abstract: In 1983, Cohen and Lenstra conjectured the distribution of p-class groups of imaginary quadratic fields (p odd). These groups are the abelianizations of p-class tower groups, whose distribution Bush, Hajir, and I recently conjectured. Wood and I then computed the moments of this distribution and proved some function field analogues.

Elena Fuchs (UC-Davis)

Title: *Thin groups and expander graphs in number theory*

Abstract: Finite index subgroups of $\mathrm{SL}(2,\mathbb{Z})$ have long been studied from the number theoretic point of view, for example as the symmetry groups underlying modular forms. However, arithmetic problems involving infinite index or thin subgroups have only recently become approachable due to groundbreaking results in the past decade concerning families of expander graphs. Still, this area of number theory is only in its infancy. In this talk, we discuss the history and recent results concerning the dichotomy of thin vs non-thin groups, and present work on the arithmetic of rather general families of thin groups.

Arul Shankar (University of Toronto)

Title: *Polynomials with square-free discriminant*

Abstract: A classical question in analytic number theory is that of determining the density of squarefree values taken by an integer multivariable polynomial. The case of discriminant polynomials of single variable polynomials is particularly interesting from the point of view of arithmetic statistics. I will describe joint work with Manjul Bhargava and Xiaoheng Wang, in which we prove that for all n greater than v1, a positive proportion of monic integer polynomials of degree n have squarefree discriminant

Invited Postdoctoral Speaker

Efrat Bank (University of Michigan)

Title: *Primes in short intervals on curves over finite fields*

Abstract: We prove an analogue of the Prime Number Theorem for short intervals on a smooth proper curve of arbitrary genus over a finite field. Our main result gives a uniform asymptotic count of those rational functions, inside short intervals defined by a very ample effective divisor E, whose principal divisors are prime away from E. In this talk, I will discuss the setting and definitions we use in order to make sense of such count, and will give a rough sketch of the proof. This is a joint work with Tyler Foster.

Invited Graduate Student Speaker

Alexandra Florea (Stanford University)

Title: *The 4th moment of quadratic Dirichlet L-functions in function fields.*

Abstract: I will discuss moments of L?functions over function fields, and I will focus on the fourth moment in the family of quadratic Dirichlet L?functions. Summing $L(1/2, \chi_? D)$ over monic, square-free polynomials D of degree $2g+1$ with coefficients in the fixed finite field $\mathbb{F}_q$, Andrade and Keating conjectured that the fourth moment is asymptotic to $q^{2g+1}P(2g+1)$, where P is a polynomial of degree 10. I will explain how one can obtain an asymptotic formula for the fourth moment with 3 of the leading terms, as the genus $g \to \infty$.

Contributed Talks

Lea Beneish (graduate student, Emory University)

Title: *Moonshine modules and a question of Griess*

Abstract: We consider the situation in which a finite group acts on an infinite-dimensional graded module in such a way that the graded-trace functions are weakly holomorphic modular forms. Under some mild hypotheses we completely describe the asymptotic module structure of the homogeneous subspaces. As a consequence we find that moonshine for a group gives rise to partial orderings on its irreducible representations. This serves as a first answer to a question posed by Griess.

Stevo Bozinovski (South Carolina State University)

Title: *A property of Riemann zeta function in the context of Analytic Number Theory*

Abstract:

Theorem: Riemann-zeta(-1) = Dirichlet-eta(-2) if their corresponding Dirichlet series have same odd number of elements. This result was obtained in 2015 and presented at PANTS XXVI in Greensboro, NC, in 2016. A continuation of this line of work is a consideration of this result in the context of Analytic Number Theory. This work is a collaboration with Adrijan Bozinovski.

Luis Finotti (University of Tennessee)

Title: *Weierstrass Coefficients of the Canonical Lifting*

Abstract: The Weierstrass coefficients of the canonical lifting of an ordinary elliptic curve can be given as functions on the Weierstrass coefficients of the original curve. We will describe these functions. In particular, we will show that, when seeing these coefficients as Witt vectors, their entries can be given by universally defined modular functions.

Bobby Grizzard (University of Wisconsin)

Title: *Slicing the Stars*

Abstract: There will be pictures. We'll discuss the problem of counting the number of algebraic numbers of given degree and bounded height, as the height bound grows. For general algebraic numbers, this was done by Masser and Vaaler, and for algebraic integers by Barroero. Since we count algebraic numbers by counting their minimal polynomials, the problem becomes that of counting lattice points in a certain "star body" whose volume was computed by Chern and Vaaler. We'll talk about how to count algebraic units and more by carefully counting lattice points in "slices" of these star bodies. This is joint work with Joseph Gunther (CUNY).

Wilson Harvey (University of South Carolina)

Title: *Covering Thin Subsets of the Integers*

Abstract: We consider two famous questions about covering systems of the integers and the analogous questions for covering subsets of the integers. In particular, we show that the Fibonacci numbers can be covered by congruences with the moduli arbitrarily large and with arbitrarily large prime divisors. One consequence of this is the existence of an odd covering of the Fibonacci numbers. If there is time, we will also show that there are arbitrarily thin sets that, if covered, yield a covering of the entire set of integers.

Jesse Kass (University of South Carolina).

Title: *A classical proof that the algebraic homotopy class of a rational function is the residue pairing*

Abstract: C. Cazanave has identified the algebraic homotopy class of a rational function of 1 variable with an explicit nondegenerate symmetric bilinear form. Here we relate Cazanave's result to classical results and in particular identify Cazanave's form with a residue pairing from commutative algebra. We announce work identifying the local A1-degree of a polynomial map in several variables that has an isolated zero at the origin with the residue pairing.

Jackson Morrow (Emory University)

Title: *Selmer groups of elliptic curves over K with K-rational torsion points*

Abstract: The problem of constructing elements in the Selmer group of an elliptic curve E defined over a number field K is a classical question with many avenues of approach. In this talk, we will consider the role the torsion subgroup of E/K plays in this construction, with a particular focus on the case when E/K has a K-rational torsion point of odd, prime order. In 1988, Frey provided explicit examples of quadratic twist of elliptic curves over $\mathbf{Q}$ with $\mathbf{Q}$-rational points of odd, prime order p whose p-Selmer groups are non-trivial. We will discuss a generalization of Frey's result to elliptic curves defined over number fields of small degree as well as some possible directions for future work.

I will state and prove a new identity which connects theta series associated with binary quadratic forms of idoneal discriminants Δ and Δp^2, for p a prime. I then illustrate how to use this identity to derive Lambert series identities and hence product representation formulas for certain forms. Last but not least, I discuss generalizations to non-idoneal discriminants and the resulting theta series identities one may derive.

Kate Thompson (DePaul University)

Title: *Almost Universal Quadratic Forms*

Abstract: The study of universal quadratic forms has long been of interest to number theorists. Quadratic forms that are "almost universal," failing to represent some finite set of numbers, are less explored, though Halmos (1938) published a list of diagonal quaternary forms that except just one value. Using analytic and computational methods from Bhargava and Hanke, we provide an enumeration of and proofs for all possible pairs m, n and possible singleton values m that a classically integral, positive-definite, quadratic form may except.

Ali Uncu (University of Florida)

Title: *Weighted Partition Results Inspired by Nathan Fine's False Theta Identities*

Abstract: We utilize results of Nathan Fine to discover new partition identities involving weights. These relations connect Gollnitz-Gordon type partitions and partitions with distinct odd parts, partitions into distinct parts and ordinary partitions, respectively. Some of these weights are of additive nature and involve new partition statistics. One example of such statistics is the number of different odd parts of a partition larger than or equal to a given value.