Potential in a polygonal body coated with a thin dielectric layer.

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Approximate boundary conditions appear in many applications in electromagnetism : anechoïd chambers, wave absorption by dielectric layers. In such situations, in order to determine the field, we need to solve a transmission problem in a domain - bounded or not - with a thin dielectric layer of thickness $\varepsilon \ll 1$. For the numerical treatment, a fine mesh is needed, the elements being of size ε : the computation becomes very long and may not be accurate. That leads us to repleace the thin layer by a boundary condition, called *approximate boundary condition* or *impedance condition*. In the new problem - which is close to the original one -, the thin layer does not appear anymore and we can use a coarser mesh.
The obtention of such boundary conditions is well known in the case of smooth domains (see [1]) : it is based on the construction of an asymptotic expansion of the solution as $\varepsilon \to 0$, obtained via a dilatation of the thin layer in the normal direction. We are here interested in a two-dimensional situation of a corner domain. Singularities appear near the corner and we cannot perform the same technic as in the smooth case.
Our method consists in a decomposition of the solution into regular and singular parts ; the first one can be studied via the "smooth techniques". But the second one requires tools coming from the theory of singularities in corner domains (Mellin transform, see [2]). We introduce an auxilliary problem in an infinite sector, which allows the construction of *profiles* which are the basis of the structure of the expansion of the singular part near the corner. The complete expansion involves non integer exponents of ε, depending on the opening of the corner. The impedance conditions are not as efficient as in the smooth case : the nearest the angle is from 2π, the less precise is the approximation of the thin layer by the impedance condition.
Numerical tests have been performed with the library MÉLINA. The illustration of the asymptotic expansion needs very accurate results, allowed by high order elements. The $p-$version is particularly adapted to our case because we can use anisotropic elements in the layer.

References

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