Corner singularities in 3D: Numerical computation and applications

A. Dimitrov

Institute of Solid Mechanics, Karlsruhe University, P.O. Box 69 80, 76128 Karlsruhe, Germany
e-mail: Atanas.Dimitrov@mach.uni-karlsruhe.de

ABSTRACT

It is well known that the solution of an elliptic boundary value problem may contain gradient singularities in a non-smooth domain. In the case of cones or polyhedral corners in $\mathbb{R}^3$ the solution behaves in the vicinity of the singular point asymptotically like

$$\mathbf{u} = \sum_i \sum_{k=0}^{k_j} K_{ik} \, |\mathbf{x}|^{\lambda_i} \, \ln^k(|\mathbf{x}|) \, \mathbf{U}_{ik}(\mathbf{x}/|\mathbf{x}|), \tag{1}$$

where λ_i are the singularity exponents, $\mathbf{U}_{ik}$ the so called angular functions and K_{ij} the corner stress intensity factors (CSIFs), see for instance [1] and the references therein. If one considers special geometries or material properties the series (1) can be constructed explicitly. However, for general three-dimensional problems some numerical methods are needed.

In this talk we present a general numerical procedure for the computation of corner singularities in $\mathbb{R}^3$ for compressible elasticity and Stokes flow problems [2–4]. The method is based on a weak formulation and a Galerkin-Petrov finite-element approximation. In the case of Stokes flow problems (or incompressible elasticity) a mixed $\mathbf{u}, p$ formulation is used. In both cases an algebraic eigenvalue problem from type

$$(\mathbf{P} + \lambda \mathbf{Q} + \lambda^{\mathbf{2}} \mathbf{R}) \, \mathbf{d} = \mathbf{0}, \tag{2}$$

is obtained, which depends only on $\mathbf{x}/|\mathbf{x}|$ and not on $|\mathbf{x}|$, due to a separation of variables. Thus only a part of the unit sphere should be discretized. For the numerical solution of (2) the iterative Arnoldi method is used and therefore only *one* direct matrix factorization as well as few matrix-vector products are needed to find *all* eigenvalues $\Re(\lambda) \in (-0.5, 1.0)$ as well as the corresponding eigenvectors simultaneously. In the case of elastic problems with homogeneous material properties also an adaptive remeshing technique based on *a-posteriori* error estimation is provided. Some benchmark tests show, that this method is robust and very accurate.

In the last part of the talk several numerical results are presented for illustrating the applicabillity to problems of practical interest [2–6]. The results for the surface-breaking crack for instance are used to explain some 3D-effects in crack propagation detected by many numerical and experimental findings in the framework of linear elastic fracture mechanics [6].

References

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