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Math 374 – Homework 1 Due September 6 **Name:**

Problems Assigned: **1.1:** 6, 10, 11abef, 13, 16, 17ei, 21;
1.2: 3, 6, 11, 17, 22, 36, 43, 44.

1.1.6. Several forms of negation are given for each statement. Which are correct?

a. The carton is sealed or the milk is sour.
1. The milk is not sour or the carton is not sealed.
2. The carton is not sealed and also the milk is not sour.
3. If the carton is not sealed, then the milk will be sour.

b. Flowers will bloom only if it rains.
1. The flowers will bloom but it will not rain.
2. The flowers will not bloom and it will not rain.
3. The flowers will not bloom or else it will not rain.

c. If you build it, they will come.
1. If you build it, then they won't come.
2. You don't build it, but they do come.
3. You build it, but they don't come.

Solution:

a.

b.

c.

1.1.10. Let A, B, C, and D be the following statements:

$$\begin{aligned} A &= \text{ The villain is French.} \\ B &= \text{ The hero is American.} \\ C &= \text{ The heroine is British.} \\ D &= \text{ The movie is good.} \end{aligned}$$

Translate the following compound statements into symbolic notation.

a. The hero is American and the movie is good.
b. Although the villain is French, the movie is good.
c. If the movie is good, then either the hero is American or the heroine is British.
d. The hero is not American, but the villain is French.
e. A British heroine is a necessary condition for the movie to be good.

Solution:

a.

b.

c.

d.

e.

1.1.11abef. Let A, B, and C be the following statements:

$$\begin{aligned} A &= \text{ Roses are red.} \\ B &= \text{ Violets are blue.} \\ C &= \text{ Sugar is sweet.} \end{aligned}$$

Translate the following statements into English sentences.

a. $B \vee C'$
b. $B' \vee (A \to C)$
e. $(B \wedge C')' \to A$
f. $A \vee (B \wedge C')$

Solution:

a.

b.

e.

f.

1.1.13. Using letters H, K, A for the component statements, translate the following compound statements into symbolic notation.

a. If the horse is fresh, then the knight will win.
b. The knight will win only if the horse is fresh and the armor is strong.
c. A fresh horse is a necessary condition for the knight to win.
d. The knight will win if and only if the armor is strong.
e. A sufficient condition for the knight to win is that the armor is strong or the horse is fresh.

Solution:

a.

b.

c.

d.

e.

1.1.16. Using letters P, C, B, L for the component statements, translate the following compound statements into symbolic notation.

a. If the project is finished soon, then the client will be happy and the bills will be paid.
b. If the bills are not paid, then the lights will go out.
c. The project will be finished soon only if the lights do not go out.
d. If the bills are not paid and the lights go out, then the client will not be happy.
e. The bills will be paid if and only if the project is finished soon, or else the lights go out.
f. The bills will be paid if and only if either the project is finished soon or the lights go out.

Solution: Define

$P =$ The project is finished soon.

$C =$

$B =$

$L =$

a.

b.

c.

d.

e.

f.

1.1.17ei. Construct truth tables for the following wffs. Note any tautologies or contradictions.

e. $(A \to B) \to [(A \vee C) \to (B \vee C)]$
i. $[(A \vee B) \wedge C'] \to A' \vee C$

Solution:

e. Let P denote the wff $(A \to B) \to [(A \vee C) \to (B \vee C)]$. Then

A	B	C					P

The wff P is a ☐ tautology ☐ contradiction ☐ neither.

i. Let P denote the wff $[(A \vee B) \wedge C'] \rightarrow A' \vee C$. Then ...

The wff P is ...

1.1.21. Prove the following tautologies by starting with the left side and finding a series of equivalent wffs that will convert the left side into the right side. You may use any of the equivalencies in the list on page 8 or in Exercise 20.

a. $(A \wedge B') \wedge C \leftrightarrow (A \wedge C) \wedge B'$
b. $(A \vee B) \wedge (A \vee B') \leftrightarrow A$
c. $A \vee (B \wedge A') \leftrightarrow A \vee B$
d. $(A \wedge B')' \vee B \leftrightarrow A' \vee B$
e. $A \wedge (A \wedge B')' \leftrightarrow A \wedge B$

Solution:

a.

$$(A \wedge B') \wedge C \iff$$

$$\iff$$

$$\iff$$

b.

$$(A \vee B) \wedge (A \vee B')$$

c.

$$A \vee (B \wedge A')$$

d.

$(A \wedge B')' \vee B$

e.

$A \wedge (A \wedge B')'$

1.2.3. What inference rule is illustrated by the argument: The dog has a shiny coat and loves to bark. Consequently, the dog loves to bark.

Answer:

1.2.6. Decide what conclusion, if any, can be reached from the given hypothesis. Justify.

Either the weather will turn bad or we will leave on time. If the weather turns bad, then the flight may be canceled.

Solution:

1.2.11. Justify each step in the proof sequence of $A' \wedge B \wedge [B \to (A \vee C)] \to C$.

1. A'
2. B
3. $B \to (A \vee C)$
4. $A \vee C$
5. $(A')' \vee C$
6. $A' \to C$
7. C

1.2.17. Prove $(A' \to B') \wedge B \wedge (A \to C) \to C$ is valid.
Solution:

1.2.22. Prove $[A \to (B \vee C)] \wedge C' \to (A \to B)$ is valid.
Solution:

1.2.36. Prove $(A \vee B) \wedge (A \to C) \wedge (B \to C) \to C$ is valid.
Solution:

1.2.43. Use propositional logic to prove the following argument is valid. Use the letters shown.

The crop is good, but there is not enough water. If there is a lot of rain or not a lot of sun, then there is enough water. Therefore, the crop is good and there is a lot of sun.
C, W, R, S

Solution:

1.2.44. Use propositional logic to prove the following argument is valid. Use the letters shown.

If the ad is successful, then the sales volume will go up. Either the ad is successful or the store will close. The sales volume will not go up. Therefore, the store will close.
A, S, C

Solution: