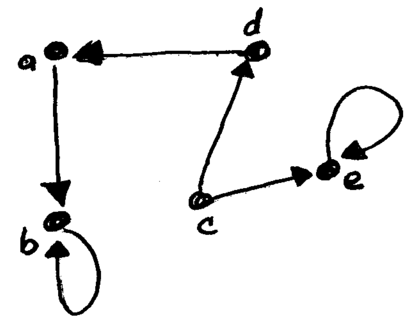
Math 374
Practice test 4

1) a) Write down the elements of the binary relation given by the illustration.



$\{(a,b), (b,b), (d,a), (c,d), (c,e), (e,e)\}$

b) Is the relation one-to-one, one to many, many to one, or many to many? Is it symmetric?

It's <u>Many</u> to <u>Many</u> it's <u>not</u> symmetric actually, its <u>anti symmetric</u>

c) What must be added to make the relation:

i) Reflexive

add $(a,a), (d,d), (c,c)$

ii) Transitive

add $(c,a), (d,b), (c,b)$

iii) Symmetric

add $(b,a), (a,d), (d,c), (e,c)$

2) If a set S has n elements in it, how many different binary relations are there on S?

$|S \times S| = n^2$, so $|\mathcal{P}(S \times S)| = \boxed{2^{n^2}}$

3) Consider these two relations on $S = \{1, 2, 3\}$

$\rho = \{(1,1), (2,2), (3,3), (1,2), (2,3)\}$

$\sigma = \{(1,3), (2,2), (3,3)\}$

a) Write the elements of ρ'.

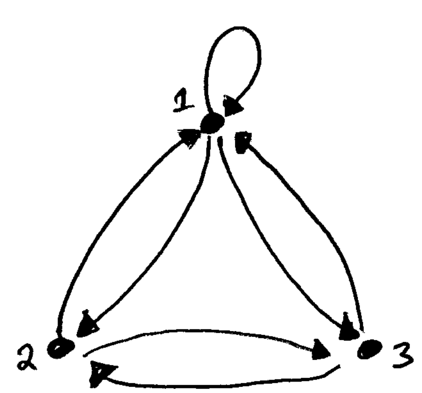
$\{(1,3), (2,1), (3,1), (3,2)\}$

b) What properties does $\rho \cup \sigma$ have? (i.e., Reflexive? Symmetric? ...)

It's Reflexive, Transitive, and Antisymmetric

(it's a Poset!!!)

c) Illustrate the relation $(\rho \cap \sigma)'$.



4) Let $S = \{1,2,3,4\}$ ad recall The poset on $\mathcal{P}(S)$ defined by $A \preceq B$ when $A \subseteq B$.

a) what are all of The minimal elements of The poset?

Just $\boxed{\emptyset}$

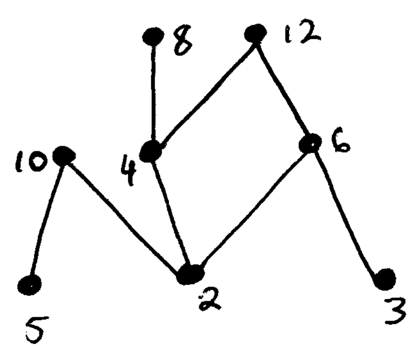
b) Write all of The predecessors of $\{2,3\}$ in the poset

$\{2\}$, $\{3\}$, $\emptyset$

c) Write all of The immediate successors of $\{2,3\}$.

$\{1,2,3\}$, $\{2,3,4\}$

5) Draw the Hasse Diagram for the poset on $S = \{2,3,4,5,6,8,9,10,12\}$ where $x \preceq y$ if x divides y.



6) The set $S = \{a, b, c, d, e, f,$ [redacted]$\}$

can be partitioned into $A_1 = \{a, c, e,$ [redacted]$\}$

$A_2 = \{b, f\}$, $A_3 = \{d\}$.

Write all of the elements of the equivalence relation that this defines.

$\{ (a,a), (b,b), (c,c), (d,d), (e,e), (f,f),$

$(a,c), (c,a), (a,e), (e,a), (c,e), (e,c),$

$(b,f), (f,b) \}$

7) Show that the relation ρ on $\mathbb{N}$ defined by $x \rho y$ if $x+y$ is <u>even</u> is an equivalence relation. (Show the <u>three</u> Properties).

Is the same true for "$x \sigma y$ if $x+y$ is divisible by three" as the relation? (Prove or show why not...)

a) <u>Reflexive.</u> $a+a = 2a$ is always <u>even</u>.

b) <u>Symmetric</u>. If $x \rho y$, Then $x+y$ is even; so is $y+x$. (duh)

So $y \rho x$.

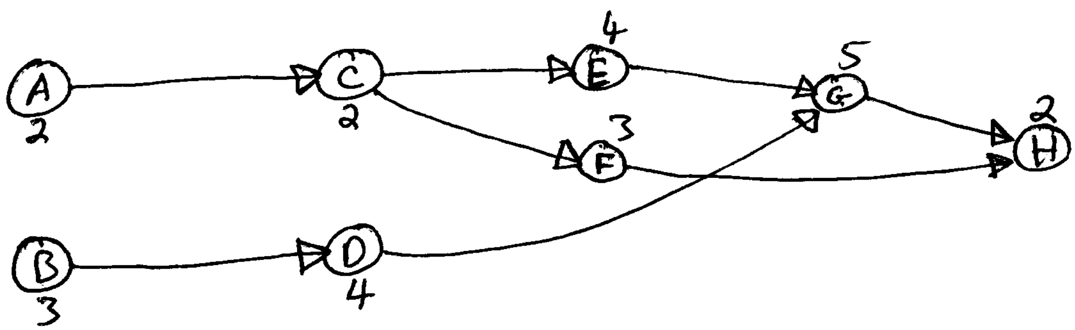
c) <u>Transitive</u> If $x \rho y$, and $y \rho z$, Then x has the same <u>parity</u> (odd or even) as y. Also, y has the same parity as z. Hence x and z have the same parity. Adding two odd or two evens always gives an even. Hence $x \rho z$.

Not true for $x \sigma y$. Note, $1 \sigma 2$, and $2 \sigma 1$. (since the sum is 3)

If it was <u>transitive</u> we have <u>$1 \sigma 1$</u> which isn't True.

8) The following is a list of activities to be done in order to install an air pollution control system. Draw the PERT Chart, and find the Critical Path, and its length.

Activity	Immediate Predecessors	Time
A) Build Components	∅	2
B) Modify roof	∅	3
C) Construct the stack	A	2
D) Pour Concrete	B	4
E) ~~Bu~~ Build Burner	C	4
F) Install control system	C	3
G) Install air device	D, E	5
H) Inspect & Test	F, G	2



Min time to complete

A	2
B	3
C	4
D	7
E	8
F	7
G	13 (through E)
H	15 (through G)

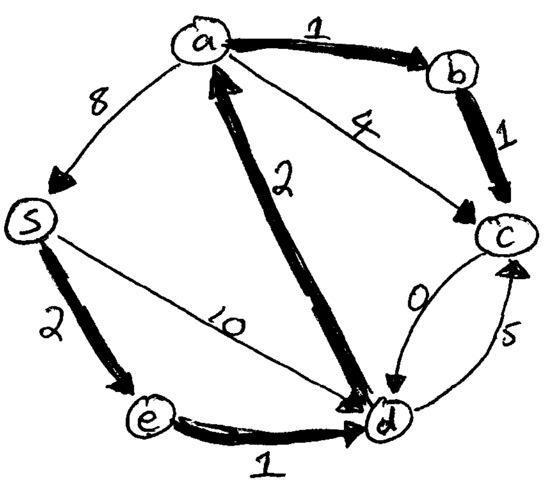
Time to complete is
15

CP is
$\boxed{ACEGH}$

9) Do a topological sort (different from Alphabetical) for the Poset in problem 8.

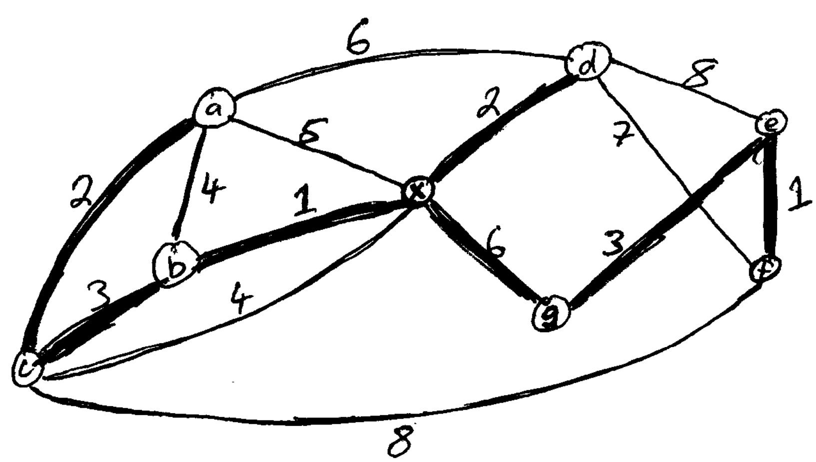
$\boxed{B, D, A, C, F, E, G, H}$

10) Find the shortest paths from $\underline{s}$ to every other vertex in the following digraph. Highlight the tree edges it defines.



	a	b	c	d	e
s	∞	∞	∞	10 s	2 s
e	∞	∞	∞	3 e	
d	5 d	∞	8 d		
a		6 a	9 d		
b			7 b		
c	(5,d)	(6,a)	(7,b)	(3,e)	(2,s)

11) Use Prim's and Kruskal's algorithms to find a MST in the following graph. List the edges in the order chosen, and start Prim's from vertex **x**.



a) Prim's

(x,b), (x,d), (b,c), (c,a), (x,g), (g,e), (e,f)

b) Kruskal's

(x,b), (e,f), (a,c), (x,d), (g,e), (c,b), (x,g)