Math 374
Practice Test 3

1) Let $A = \{1,2,3,4\}$, $B = \{a, \{b\}, \{a, \{b\}\}\}$

a) List the set $A \cup B$.

b) What is the cardinality of $A \times B$?

c) What is the cardinality of $\mathcal{P}(B)$?

d) What is the cardinality of $\mathcal{P}(\mathcal{P}(B))$?

e) For each of the following determine if the statement is true or false.

i) $\{a, \{b\}\} \subseteq B$

ii) $\{a, \{b\}\} \in B$

iii) $\{1,2\} \in A$

iv) $\{1,2\} \in \mathcal{P}(A)$

v) $\{a,b\} \subseteq B$

vi) $\emptyset \subseteq A \cap B$

vii) $\emptyset \in \mathcal{P}(A) \cap \mathcal{P}(B)$

viii) $\{\{1,2\}, \{b\}\} \subseteq A \cup B$

2) Liscence plates in Utah come in two types: Standard plates and vanity plates. Standard plates have three distinct letters, followed by 3 digits, where the first digit must be nonzero. Vanity plates use two letters (not neccesarily distinct) followed by 3 digits, where again the first digit must not be zero. How many different Liscence plates are there?

3) A standard set of pool balls is a collection of 15 balls, numbered (unsorprisingly) 1–15. Balls 1-8 are <u>solids</u>, and balls 9-15 are <u>stripes</u>.

a) How many Collections of 3 balls are there?

b) How many Collections of 5 balls have 3 stripes and 2 solids?

c) How many 5-ball collections are there with at <u>least</u> one solid?

4) We surveyed 150 college students about how they commute to school. 83 said they sometimes drive, 97 said they sometimes bike, and 28 said they sometimes ride a skateboard. 53 bike and drive, 14 drive and skateboard, and 7 bike and skateboard. 2 students said they sometimes drive, sometimes bike, and sometimes ride a skateboard. How many of the surveyed students use none of these 3 modes of transport?

5) Show that in any collection of 51 different numbers, bigger than 0 and less than 100, that two of the numbers add to 100.

6) A dealership has 8 identical brand new cars to put on display. 3 are blue, 2 are red, 2 are green, and one is white. How many different ways can the dealership park them in a line?

7) I've come across 21 bitcoins, and decided to distribute them to 5 of my friends.

a) How many ways can I do this?

b) How many ways can I do this if I promise each of my 5 friends that they get at least 1 bitcoin?

8) Expand $(2x - y)^4$ using the binomial theorem

9) What is the coefficient of a^5b^6 in $\left(2a - \frac{b}{2}\right)^{11}$?