

Math 374
Practice Test 2

Prove or Disprove The following Statements:

- 1) The sum of two odd integers is even
- 2) The product of two irrational numbers is irrational.
- 3) The sum of a rational and an irrational is irrational.

4) Prove That $1 + 2^1 + 2^2 + 2^3 + \dots + 2^n = 2^{n+1} - 1$
for all $n \geq 1$.

5) Prove That The following function correctly evaluates 2^n , ~~by~~
by proving That $Q: \{j = 2^i\}$ is a loop invariant,
and evaluating The postcondition at loop termination.

Power (n): (positive integer)

$i = 1$

$j = 2$

while $i \neq n$ do

$j = j * 2$

$i = i + 1$

end while

return j

6) Give a recursive definition for all binary strings with an odd number of 0's.

7) Write the first 5 values of the sequence.

$$M(1) = 2$$

$$M(2) = 2$$

$$M(n) = 2(M(n-1)) + M(n-2) \quad \text{for } n > 2.$$

8) Find a closed formula, and prove it holds, for the following recurrence relation.

$$F(1) = 2,$$

$$F(n) = 2F(n-1) + 2^n \quad \text{for } n \geq 1.$$