Math 374

Practice Test 1

1) Write the negation of each statement

a) If the food is good, then the service is excellent.

$(F \to S)' \Leftrightarrow (F' \vee S)' \Leftrightarrow F \wedge S'$

"The food is good and the service is poor."

b) Either the food is good, or the service is excellent.

$(F \vee S)' \Leftrightarrow F' \wedge S'$

"The food is bad and the service is poor"

c) If the price is high, then the food is good and the service is excellent.

$[P \to (F \wedge S)]' \Leftrightarrow [P' \vee (F \wedge S)]' \Leftrightarrow P \wedge (F \wedge S)' \Leftrightarrow P \wedge (F' \vee S')$

"The price is high, and either the food is bad or the service is poor"

2) Using letters for the component statements, translate the following compound statements into symbolic notation

a) If prices go up, then housing will be plentiful and expensive but if housing is not expensive, then it will still be plentiful. (G = "go up", P = "plentiful", E = "Expensive")

$$[G \to (P \wedge E)] \wedge [E' \to P]$$

3) Construct a truth table for the following wff.

$$(A \vee B') \wedge (A \to B)$$

A	B	B'	$(A \vee B')$	$(A \to B)$	$(A \vee B') \wedge (A \to B)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	T	T	T

4) Construct a proof sequence for the following wff.

$$(P \vee Q') \wedge (P' \vee R) \to (Q' \vee R)$$

equivalent to (by implication)

$$(P \vee Q') \wedge (P' \vee R) \to (Q \to R)$$

equivalent to (by deduction method)

$$(P \vee Q') \wedge (P' \vee R) \wedge Q \to R$$

1) $P \vee Q'$ hyp

2) $P' \vee R$ hyp

3) Q hyp

4) $Q \to P$ imp (1)

5) P mp. (3,4)

6) $P \to R$ imp (2)

7) R mp (5,6).

5) Construct a proof sequence for the following wff.

$$A \wedge (B \to C) \wedge [(A \wedge B) \to (D \vee C')] \wedge B \to D$$

1) A hyp
2) $B \to C$ hyp
3) $(A \wedge B) \to (D \vee C')$ hyp
4) B hyp
5) $A \wedge B$ conj. (1,4)
6) $D \vee C'$ MP (5,3)
7) $C \to D$ imp 6
8) C mp (2,4)
9) D mp (7,8)

6) Write the following statements in symbolic notation where

$B(x) =$ "x is a Bear" $S(x) =$ "x is a Salmon" $E(x,y) =$ "x eats y"

a) All bears eat all salmon

$$(\forall x)(\forall y)[B(x) \wedge S(y) \to E(x,y)]$$

b) Some bears eat all salmon

$$(\exists x)(\forall y)[B(x) \wedge S(y) \to E(x,y)]$$

c) Only bears eat salmon.

$$(\forall x)(\forall y)[S(x) \wedge E(y,x) \to B(y)]$$

7) Give a proof sequence for the following valid wff.

$$(\exists x)\, P(x) \wedge (\forall x)\big(P(x) \to Q(x)\big) \to (\exists x)\, Q(x)$$

1) $(\exists x) P(x)$ hyp
2) $(\forall x)(P(x) \to Q(x))$ hyp
3) $P(a)$ e.i., (1)
4) $P(a) \to Q(a)$ u.i. (2)
5) $Q(a)$ mp, (3,4)
6) $(\exists x) Q(x)$ e.g. (5)

8) Using a domain of interpretation, show that the following wff is not valid.

$$(\forall x)\,[P(x) \to Q(x)] \to [(\exists x) P(x) \to (\forall x) Q(x)]$$

Domain is all animals. $P(x)$ = "x is a Tiger" $Q(x)$ = "x is a cat"

$(\forall x)[P(x) \to Q(x)]$ says "for all animals, if it's a Tiger, Then it's a cat", which is a true statement.

$[(\exists x) P(x) \to (\forall x) Q(x)]$ says "If There exists some tiger, then every animal is a cat", which is false.

Since The left side is true, and The right side is false, The statement is not valid.

9) Translate The following paragraph into a symbolic statement, where $O(x,y)$ = "x owns y" $F(x)$ = "x is a farmer" $C(x)$ = "x is a cow" $D(x)$ = "x is a dentist"

- Every farmer owns a cow. No dentist owns a cow. Therefore, no dentist is a farmer.

$$(\forall x)\big[F(x) \to (\exists y)(C(y) \wedge O(x,y))\big] \wedge (\forall x)\big[D(x) \to [(\exists y)(C(y) \wedge O(x,y))]'\big]$$
$$\to (\forall x)\,[D(x) \wedge F(x)]'$$

10) Show that the following program is correct, where the precondition is $P = \{x = a \text{ and } y = b \text{ and } z = c\}$ and the postcondition is $Q = \{x = c \text{ and } y = a \text{ and } z = b\}$

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temp = x
x = z
z = y
y = temp
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$\{z = c \text{ and } x = a \text{ and } y = b\}$

temp = x

$\{z = c \text{ and } temp = a \text{ and } y = b\}$

x = z

$\{x = c \text{ and } temp = a \text{ and } y = b\}$

z = y

$\{x = c \text{ and } temp = a \text{ and } z = b\}$

y = temp

$\{x = c \text{ and } y = a \text{ and } z = b\}$

All are true by assignment Rule

11) Show that the following program is correct, with given pre and postcondition

$\{x = 7\}$

<u>if</u> $x \geq 0$ then

  y = x

<u>else</u>

  y = 2x

<u>endif</u>

$\{y = 7\}$

a) show $\{x = 7 \wedge x \geq 0\}$ $y = x$ $\{y = 7\}$.

$\{x = 7\}$
$y = x$
$\{y = 7\}$

is true by assignment, and $\{x = 7\}$ is equivalent to $\{x = 7 \wedge x \geq 0\}$.

Hence our first triple holds.

b) Show $\{x = 7 \wedge x < 0\}$ $y = 2x$ $\{y = 7\}$.

Here, $\{x = 7 \wedge x < 0\}$ is always false, and an implication with a false antecedent is always true. Hence the second triple holds.

By a) and b), the conditional rule says our Program is correct.