1) Using a truth table show That

$$[(A \to B) \land (B \to C)] \to (A \to C)$$

is a tautology.

2) Prove That

$$[(A \to B) \land (B \to C)] \to (A \to C) \text{ is valid using a}$$

Proof sequence

3) Add explanations to the steps in The proof sequence for

$$A \land (B \to C) \land [(A \land B) \to (D \lor C')] \land B \to D$$

1.) A _____

2) $B \to C$ _____

3) $(A \land B) \to (D \lor C')$ _____

4) B _____

5) C _____

6) $A \land B$ _____

7) $D \lor C'$ _____

8) $C \to D$ _____

9) D _____

4) Let B(x) be "x is a bear"
   E(x,y) be "x eats y",
   S(x) be "x is a salmon".

Write the following statements, using the world as the domain.

   a) All bears eat all salmon

   b) Some bears eat all salmon

   c) Only bears eat salmon.

5) Give a proof sequence for the following argument.

$$(\exists x)\, P(x) \wedge (\forall x)(P(x) \Rightarrow Q(x)) \rightarrow (\exists x) Q(x).$$

6) Give an interpretation to show the following is __not__ valid.

$$(\forall x)\left[P(x) \rightarrow Q(x)\right] \rightarrow \left[(\exists x) P(x) \rightarrow (\forall x) Q(x)\right]$$

(7) Variables X, Y, Z have values a, b, c respectively. Prove That The following program changes Them to c, a, b respectively, using Hoare triples for proof of Correctness.

$$temp = X$$
$$X = Z$$
$$Z = Y$$
$$Y = temp.$$

(8) Verify The correctness of The following program with precondition and post-condition as given.

```
{x=7}
if x≥0 then
    Y=X
else
    y=2*X
end if
{y=7}.
```

9) Prove That The sum of two odd integers is odd.

10) Prove That the sum of an integer and its square is even.

11) Consider The following statement:

~~If G has no large odd cycle and no large odd hole~~

~~If G is Eulerian, then G has ever~~

   "If G has every vertex with even degree, then G is Eulerian."

Write
   a) The contrapositive
   b) The Converse

12) Prove that The sum of two rational numbers is rational.

13) Prove by induction:

   a) For all $n \geq 1$, $7^n - 2^n$ is divisible by 5.

   b) For all $n \geq 7$, $n^2 > 5n + 10$.

   c) For all $n \geq 1$,
   $$1 + 5 + 9 + \ldots + (4n-3) = n(2n-1)$$

14) Use The Euclidean algorithm to find the Greatest Common Divisor of 735 and 90.