By induction: Base Cases: $n = 7, 8$.

2.4.26 $T(7) = 2(6) + 2 = 14 \geq 2 \cdot 7$

$T(8) = 2(10) + 2 = 22 \geq 16 = 2 \cdot 8$.

Inductive step: Let $n \geq 9$. Assume $T(k) \geq 2k$ for all $7 \leq k < n$. Then

$$\begin{aligned} T(n) &= 2(T(n-2)) + 2 && \text{by Recursion} \\ &\geq 2(2(n-2)) + 2 && \text{by induction } (k = n-2). \\ &= 4n - 6 \\ &= 2n + (2n-6) \\ &\geq 2n. && (\text{Since } n \geq 9,\ 2n-6 \geq 0) \end{aligned}$$

Hence the induction holds, and so $T(n) \geq 2n$ for all $n \geq 7$.

---

2.4.43 a, b, and d are in W, c and e are not.

---

2.4.52 If $n = 0$, $n! = 1$.

If $n > 0$, $n! = n \cdot (n-1)!$

---

2.5.6 $c = 2$, $g(n) = 2^n \cdot n$, $P(1) = 2$.

By formula,

$$P(n) = 2^{n-1} \cdot 2 + \sum_{i=2}^{n} 2^{n-i} (2^i \cdot i)$$

$$= 2^n \left[ 1 + 2 + \dots + n \right] = \boxed{2^n \cdot \frac{(n+1)n}{2}}$$

2.5.22 $t^2 = 6t - 8 \rightarrow t^2 - 6t + 8 = 0$

$\rightarrow (t-4)(t-2) = 0$

$t = 4, 2.$

~~$7 = 4p +$~~ $7 = p + q$ $\rightarrow$ $4 = 2p$

$18 = 4p + 2q$ $\quad 2 = p \rightarrow q = 5$

$$\underline{A(n) = 2 \cdot 4^{n-1} + 5 \cdot 2^{n-1}}$$

---

2.5.27 $t^2 = 4t - 4 \rightarrow t^2 - 4t + 4 = 0$

$(t-2)^2 = 0$

$t = 2.$

$3 = p$ $\rightarrow$ $8 = 2q$

$14 = 2p + 2q$ $\quad 4 = q$

$$\underline{B(n) = 3 \cdot 2^{n-1} + 4(n-1)2^{n-1}}$$

---

3.1.7 a) $\{4, 6\}$

b) $\{1, 2, 3\}$

c) $\{1, 3, 5, 7, \ldots\} = \{x \mid x \text{ is an odd number}\}.$

---

3.1.10 a. F b. T c. F d. T e. T f. T

---

3.1.24 $P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$

3.1.33 a) binary operation

b) No. $0 \circ 0 = -1 \notin \mathbb{N}$. (fails closure)

c) binary operation

d) No. $\ln(x)$ is undefined for $x \leq 0$.

e) Unary operation

f) No. fails closure $3 \circ 3 = 5 \notin S$.

g) No. Not well defined. What is the output for $\frac{1}{3} \circ \frac{2}{3}$?
Both have same denominator...

h) Binary operation.

3.1.46 a) $C'$

b) $B \cap D$

c) $A \cap B$

d) $A \cap D'$

e) $D' \cap B'$

f) $C \cap A'$

g) $C \cup D$

3.3.11 Don't trust them.
They say that

$$450 = 425 + 397 + 340 - 284 - 315 - 219 + 147$$

Says

$$450 = 491.$$

3.2.23

$$10 \cdot 7 \cdot 3 \cdot 2 \cdot 2 \cdot 2$$

Color, int color, trans, AC, steering, option package.

3.3.4 A = Paint defect
B = Packaging defect
C = electronics defect.

$|A \cup B \cup C| = 40$.

$|A| = 28$, $|B| = 17$, $|C| = 13$

$|A \cap B| = 6$ $|B \cap C| = 7$ $|A \cap C| = 10$.

Want: $|A \cap B \cap C|$. By inclusion exclusion,

$$40 = 28 + 17 + 13 - 6 - 7 - 10 + |A \cap B \cap C|$$

$$40 = 35 + |A \cap B \cap C|$$

$$5 = |A \cap B \cap C|.$$

$\boxed{\text{5 parts had all 3 defects}}$

3.3.18 27.

3.3.23 Yes. Let the months be pigeonholes. If ~~more~~ no month has more than 2, there are $\leq 2 \cdot 12 = 24$ pigeons.

3.4.4
8! Permutations. $3 \cdot 7!$ of them end in a vowel.
(choose vowel to end, order the rest of the letters)

3.4.19 $\binom{21}{4} \cdot \binom{11}{3}$.

3.4.29 Pair denom · Pair · 3 other denom · Suit, suit, suit.

$$\boxed{13 \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot 4^3}$$

~~3.4.29~~
~~Pair denom · Pair · New Denom · Card · New Denom · Card · New Denom · Card~~

$$\boxed{\cancel{13 \cdot \binom{4}{2} \cdot 12 \cdot 4 \cdot 11 \cdot 4 \cdot 10 \cdot 4}}$$

3.4.30
Pair denoms · Pair · Pair · Card, diff denom

$$\binom{13}{2} \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 44$$

3.4.31
3 of Kind denom · triple · 2 other denoms · Suit · Suit.

$$13 \cdot \binom{4}{3} \cdot \binom{12}{2} \cdot 4 \cdot 4$$

3.4.32
First card · Suits for all 5 cards.

$$\boxed{10 \cdot 4^5}$$

A, 2, 3, 4, 5, 6, 7, 8, 9, 10.
Jack can't start a straight

3.4.33
Suit · 5 denominations

$$4 \cdot \binom{13}{5}$$

3.4.74 a) 8 stars determine the Safes. 2 bars separate them into 3 collections (Bidder 1, 2, or 3)

$$* \; * \mid * \; * \; * \; * \; * \mid *$$

Means 2 for Bidder 1, 5 for bidder 2, 1 for bidder 3.

Given 10 slots, there are $\binom{10}{2}$ ways of placing bars. So there are $\binom{10}{2}$ possible outcomes.

b) Same concept, but now 7 stars and 1 bar, to see who gets the remaining 7 safes.

$\binom{8}{1}$ ways.

---

3.6.1ad $(a-4)^4 = a^4 + \binom{4}{1} a^3(-4)^1 + \binom{4}{2} a^2(-4)^2 + \binom{4}{3} a(-4)^3 + (-4)^4$

$= a^4 - 16a^3 + 96a^2 - 256a + 256$

---

3.6.6 last term is $(-3y)^8 \cdot \binom{8}{8} = \underline{6305y^8}$

---

3.6.14 $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2}$

$= \binom{n}{r} + \binom{n}{r-1} + \binom{n}{r-1} + \binom{n}{r-2}$

$= \binom{n+1}{r} + \binom{n+1}{r-1}$ Pascal's formula x2

$= \binom{n+2}{r}$. Pascals formula.