Math 141-005/006 Name _____
Exam 2

Answer each problem completely and show all work in the space provided to get full credit. You may use the back of the page, but make a note of it. Carefully read the directions for each problem.

Problem 1. Complete the following differentiation formulas. Here, c and n are constants.

(1) $\frac{d}{dx}[c] =$

(2) $\frac{d}{dx}[e^x] =$

(3) $\frac{d}{dx}[\sin(x)] =$

(4) $\frac{d}{dx}[\tan(x)] =$

(5) $\frac{d}{dx}[\sec(x)] =$

(6) $\frac{d}{dx}[\arcsin(x)] =$

(7) $\frac{d}{dx}[\arctan(x)] =$

(8) $\frac{d}{dx}[\text{arcsec}(x)] =$

(9) $\frac{d}{dx}[x^n] =$

(10) $\frac{d}{dx}[\ln(x)] =$

(11) $\frac{d}{dx}[\cos(x)] =$

(12) $\frac{d}{dx}[\cot(x)] =$

(13) $\frac{d}{dx}[\csc(x)] =$

(14) $\frac{d}{dx}[\arccos(x)] =$

(15) $\frac{d}{dx}[\text{arccot}(x)] =$

(16) $\frac{d}{dx}[\text{arccsc}(x)] =$

Problem 2. Find the equation for the line tangent to the curve $y = \sin(x^2)$ at the point $x = \sqrt{\pi}$.

Problem 3. Differentiate the following with respect to x

(1) $f(x) = \ln(\sin^2(x))$

(2) $g(x) = (x^{1/2} - 12)^4 \arctan(x)$

Problem 4. Find the intervals of increase and decrease, and any local maxima and minima, for the function

$$g(x) = \frac{x^3}{x^3 + 1}.$$

Problem 5. Air is being added to a spherical balloon at a constant rate of $5\text{cm}^3/\text{min}$. At what rate is the radius growing when the volume is 36π cm^3? Recall that the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

Problem 6. Find $\frac{dy}{dx}$.

$$2x^2 - 4xy + 2y^2 = 10$$

Problem 7. Find $\frac{dy}{dx}$.

$$y = \frac{e^{4x^2}\cos(x)}{(x^3-1)^2}$$

Problem 8. Find the following limits

(1) $\lim_{x \to 0} \dfrac{e^{4x} - 1}{\sin x}$

(2) $\lim_{x \to \infty} (1/x)^{1/x}$

Problem 9. The function $f(x) = \sqrt{64 - x^2}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-8, 8]$. Find a value of c inside the interval that the theorem guarantees.

Problem 10. Using the information given about the function $f(x)$, fill in the blanks with the desired information, and use it to sketch a graph of f. Be sure to label any important points, and draw any asymptotes.

Information about f:

- Domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
- $f(0) = 2$, $f(1) = 0$, $f(-4) = 1$, $f(4) = -2$, $f(5) = -1.5$.
- $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 0$
- $\lim_{x \to -2} f(x) = \infty$ and $\lim_{x \to 2} f(x) = -1$
- $f'(x)$ is positive on $(-4, -2) \cup (4, \infty)$, and negative on $(-\infty, -4) \cup (-2, 4)$
- $f'(x)$ is increasing on $(-\infty, 5)$, and decreasing everywhere else.

(1) Intercepts _____

(2) Hor. Asymptotes _____

(3) Vert. Asymptotes _____

(4) Increasing _____

(5) Decreasing _____

(6) Local Maxs _____

(7) Local Mins _____

(8) Concave up _____

(9) Concave dn _____

(10) Inflection pts _____