PT2 Answers (Partial Solutions)

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2) a) $\infty^{\infty} = \underline{\infty}$ b) $0^0 = $ <u>Indeterminate</u>

c) $\infty - \infty = $ <u>Indeterminate</u> d) $\frac{0}{\infty} = \underline{0}$

3) a) $\frac{dy}{dx} = \boxed{3(x^4 - 4x^2 + 5)^2(4x^3 - 8x)}$

b) $\frac{dy}{dx} = \frac{(1-t^2) - t(-2t)}{(1-t^2)} = \boxed{\frac{1+t^2}{1-t^2}}$

c) $\frac{dy}{dx} = \frac{x^2 \cdot e^{1/x} \cdot \left(-\frac{1}{x^2}\right) - e^{1/x} \cdot 2x}{x^4} = \boxed{\frac{-e^{1/x}(1+2x)}{x^4}}$

d) $\frac{dy}{dx} = \boxed{\frac{4 \arcsin(2x)}{\sqrt{1-4x^2}}}$

e) $\frac{dy}{dx} = \boxed{\frac{2}{x} + 1}$

f) $\cos(xy) \cdot \left(y + x\frac{dy}{dx}\right) = 2x - \frac{dy}{dx}$ so

$$\boxed{\frac{dy}{dx} = \frac{2x - y\cos(xy)}{1 + x\cos(xy)}}$$

g) $\ln(y) = 4\ln(x^2+1) - 3\ln(2x+1) - 5\ln(3x-1)$

so

$$\frac{y'}{y} = \frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1}, \text{ so}$$

$$y' = \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}\left[\frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1}\right].$$

h) $$y' = -e^{\cos(x)}\sin(x) + \frac{1}{(1+(\arcsin(x))^2)}\cdot\frac{1}{\sqrt{1-x^2}}$$

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4) $$2x + 4x\frac{dy}{dx} + 4y + 2y\frac{dy}{dx} = 0$$

at $(2,1)$,

$$4 + 8\frac{dy}{dx} + 4 + 2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-4}{5}.$$

T.L. is $y-1 = \frac{-4}{5}(x-2)$ or $\boxed{y = \frac{-4}{5}x + \frac{13}{5}}$

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5) a) $$\frac{dV}{dt} = \frac{\pi r^2}{3}\cdot\frac{dh}{dt}$$

So $\frac{dv}{dt} = 6\pi$ $m^3/min.$

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b) $$\frac{dV}{dt} = \frac{\pi h}{3}\cdot 2r\frac{dr}{dt}$$

So $\frac{dv}{dt} = 20\pi$ $m^3/min.$

6) a) $f'(x) = 6x - 12.$

critical at $x = 2.$

x	$f(x)$
0	5
2	-7
3	-4

Max is $\underline{5}$

Min is $\underline{-7}$.

b) $f'(x) = \dfrac{x^2+1 - 2x^2}{x^2+1} = \dfrac{1-x^2}{x^2+1}$ critical at $x = \underline{1}$. (also -1)

x	$f(x)$
0	0
1	$1/2$
2	$2/5$.

Min is $\underline{0}$

Max is $\underline{1/2}$.

c) $f'(x) = e^{-\frac{x^2}{8}} - \frac{x^2}{4}e^{-\frac{x^2}{8}} = e^{-\frac{x^2}{8}}\left(1 - \frac{x^2}{4}\right)$ critical at $\underline{x = 2}$ (also -2)

x	$f(x)$	
-1	$-e^{-1/8}$	
2	$2e^{-1/2}$	≈ 1.213
4	$4e^{-2}$	$\approx .541$

Min is $-e^{-1/8}$

Max is $2e^{-1/2}$.

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7) $f'(x) = 3x^2 + 1$

Want all c so that $f'(c) = \dfrac{9+1}{2-0} = 5.$ (in $[0,2]$)

So
$$3c^2 + 1 = 5$$
$$c^2 = \frac{4}{3}$$
$$c = \pm\frac{2}{\sqrt{3}}$$
only the positive is in $[0,2]$.

$$\boxed{c = \frac{2}{\sqrt{3}}}$$

8) a) $\lim_{x \to 1} \frac{x^2-1}{x^2-x} = \lim_{x \to 1} \frac{x+1}{x} = \boxed{2.}$

b) $\lim_{x \to 2} \frac{x^2-1}{x^2-x} = \boxed{\frac{3}{2}}$

c) $\lim_{t \to 0} \frac{e^{3t}-1}{t}$ Indet type $\frac{0}{0}$.

$\overset{\boxed{L'H}}{=} \lim_{t \to 0} \frac{3e^{3t}}{1} = \boxed{3}$

d) $\lim_{x \to \infty} \frac{\ln(\ln(x))}{x}$ Indet type $\frac{\infty}{\infty}$

$\overset{\boxed{L'H}}{=} \lim_{x \to \infty} \frac{\frac{1/x}{\ln(x)}}{1} = \lim_{x \to \infty} \frac{1}{x\ln(x)} = \boxed{0}$

e) $\lim_{x \to -\infty} x^2 e^x$ Indet type $\infty \cdot 0$

$= \lim_{x \to -\infty} \frac{x^2}{e^{-x}}$ Indet type $\frac{\infty}{\infty}$.

$\overset{\boxed{L'H}}{=} \lim_{x \to -\infty} -\frac{2x}{e^{-x}}$ Indet Type $\frac{\infty}{\infty}$.

$\overset{\boxed{L'H}}{=} \lim_{x \to -\infty} \frac{2}{e^{-x}} = \frac{2}{\infty} = \boxed{0}$

f) $\lim_{x\to\infty}(x-\ln(x)) = \lim_{x\to\infty} x\left(1-\frac{\ln(x)}{x}\right)$

$= \left(\lim_{x\to\infty} x\right)\left(\lim_{x\to\infty}\left(1-\frac{\ln(x)}{x}\right)\right)$ (unless this is indeterminant)

Look at the second limit. $\lim_{x\to\infty}\frac{\ln(x)}{x}$ Indet type $\frac{\infty}{\infty}$.

$\stackrel{\boxed{L'H}}{=} \lim_{x\to\infty}\frac{1/x}{1} = 0.$

So $\lim_{x\to\infty}\left(1-\frac{\ln(x)}{x}\right) = 1.$

So $\lim_{x\to\infty} x\left(1-\frac{\ln(x)}{x}\right) = \boxed{\infty}$

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g) $\lim_{x\to 0^+}(4x+1)^{\cot(x)}$ Ind type 1^∞.

$= e^{\lim_{x\to 0^+}\cot(x)\cdot\ln(4x+1)}$

$= e^{\lim_{x\to 0^+}\frac{\ln(4x+1)}{\tan(x)}}$ ← Ind type $\frac{0}{0}$.

$\stackrel{\boxed{L'H}}{=} e^{\lim_{x\to 0^+}\frac{\frac{4}{4x+1}}{\sec^2(x)}} = \boxed{e^4}$

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h) $\lim_{x\to 0^+}(\tan(2x))^x$ Ind type 0^0

$= e^{\lim_{x\to 0^+}\frac{\ln(\tan(2x))}{x^{-1}}}$ ← Indet type $\frac{\infty}{\infty}$.

$= e^{\lim_{x\to 0^+}\frac{\frac{2\sec^2(2x)}{\tan(2x)}}{-x^{-2}}} = e^{\lim_{x\to 0^+}\frac{-2x^2}{\sin(2x)\cos(2x)}}$ Type 0/0 $\stackrel{\boxed{L'H}}{=} e^{\lim_{x\to 0^+}\frac{-2x}{\cos^2(2x)-\sin^2(2x)}}$

$= e^0 = \boxed{1}$

9) a) $y = x^3 + 6x^2 + 9x$

$D = \mathbb{R}$. No Asymptotes. ($\lim_{x\to\infty} y = \infty$, $\lim_{x\to -\infty} y = -\infty$).

$y = x(x+3)^2$, so $(0,0)$ $(-3,0)$ are intercepts.

$y' = 3x^2 + 12x + 9 = 3(x^2+4x+3) = 3(x+1)(x+3)$

so $\boxed{-1, -3}$ are critical.

		-3		-1	
$f'(x)$	$(+)$		$(-)$		$(+)$

Decrease $(-3, -1)$

Increase $(-1, \infty) \cup (-\infty, -3)$

So there is a minimum at $(-1, f(-1)) = (-1, -4)$
and maximum at $(-3, f(-3)) = (-3, 0)$

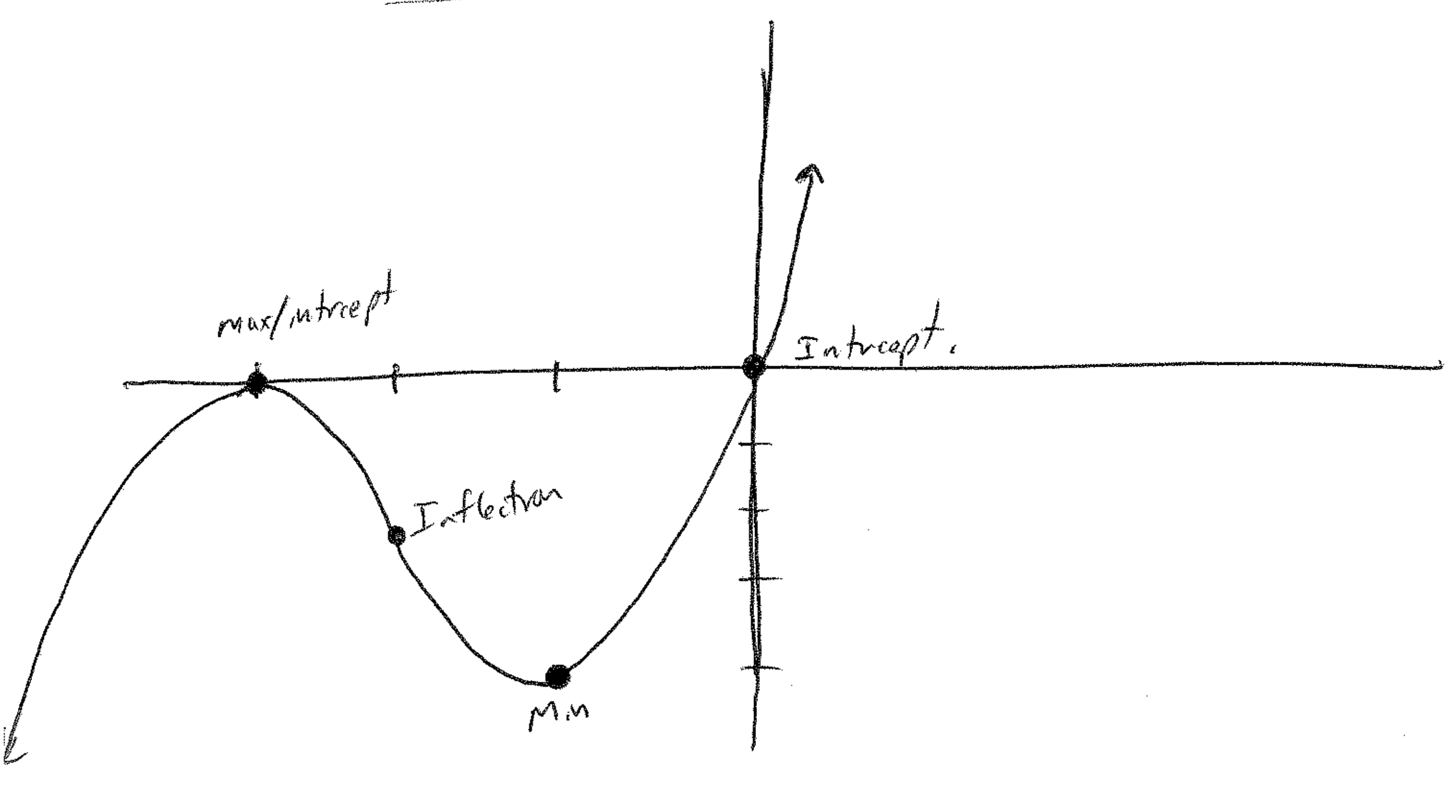
$y'' = 6x + 12$ so $y'' = 0$ at $x = -2$.

		-2	
$f''(x)$	$(-)$		$(+)$.

C-up $(-2, \infty)$

C-down $(-\infty, -2)$

Inflection at $(-2, f(-2)) = (-2, -2)$.



b) $y = \dfrac{x^2-4}{x^2-2x}$ $\quad D$ = All reals except $x=0$, $x=2$.

<u>Intercepts</u> <u>y intercept</u> (DNE, $x=0$ not in Domain)

<u>x ints</u> $x = 2, -2$, (only $x=-2$ is in the domain).

<u>Note</u> $y = \dfrac{x+2}{x}$ everywhere except $x=2$, where undefined.

<u>Asymptotes</u> $\lim\limits_{x\to 2} \dfrac{x^2-4}{x^2-2x} = \lim\limits_{x\to 2} \dfrac{x+2}{x} = 2$. No vertical at $x=2$. (hole)

$\lim\limits_{x\to 0} \dfrac{x^2-4}{x^2-2x} = \lim\limits_{x\to 0} \dfrac{x+2}{x} \to$ $-\infty$ on left, $+\infty$ on Right.

<u>VA</u> at $x=0$.

$\lim\limits_{x\to\infty} \dfrac{x^2-4}{x^2-2x} = \lim\limits_{x\to\infty} \dfrac{1-\frac{4}{x^2}}{1-\frac{2}{x}} = 1$

(same for $x \to -\infty$). <u>HA</u> at $y = 1$

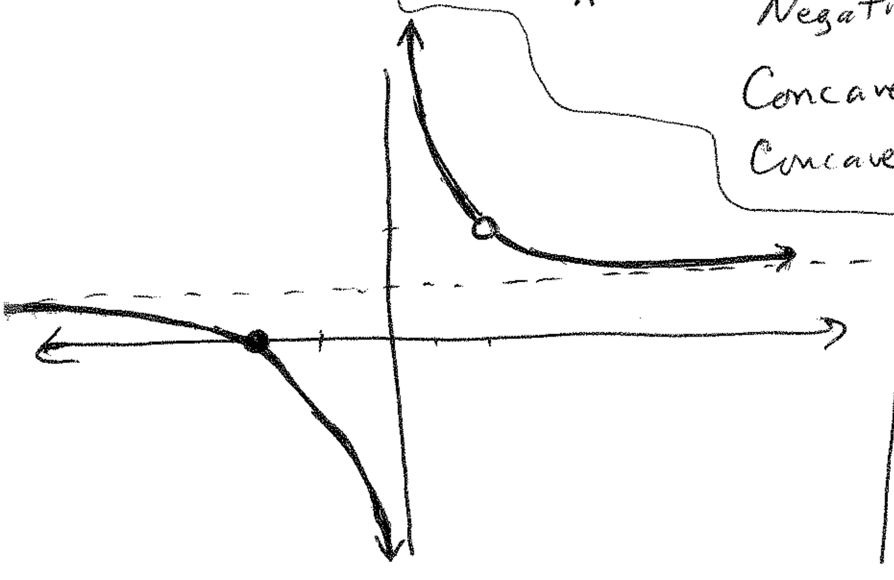
$y = 1 + \dfrac{2}{x} \to$ $y' = -\dfrac{2}{x^2}$ Critical at <u>zero</u>, always <u>Negative</u> $\boxed{\text{Decreasing } (-\infty, \infty)}$.

$y'' = \dfrac{4}{x^3}$ ~~crit~~ critical at zero.

Negative for $x<0$, Positive $x>0$

Concave <u>down</u> $(-\infty, 0)$

Concave <u>up</u> $(0, \infty)$



For c) and d), see examples 3, 5 in the text. Pg 311-312