1) $f \circ g(x) = f(g(x)) = f(x^6 - x^4) = \sqrt{x^6 - x^4}$

$= x^2 \sqrt{x^2 - 1}$.

$x^2 - 1 \geq 0$ says $x \geq 1$ or $x \leq -1$.

Domain is $(-\infty, -1] \cup [1, \infty)$.

$g \circ f(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^6 + (\sqrt{x})^4$

$= x^3 - x^2$.

Domain is $[0, \infty)$, since domain of $f(x)$ is.

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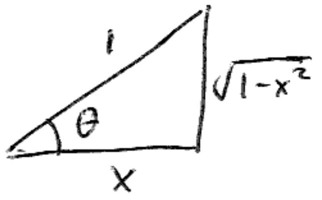
2) $\sin(\arccos(x))$



$\arccos(x) = \theta$

$\sin(\theta) = \sqrt{1 - x^2}$

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3) a) $\ln\left(\frac{2}{t}\right) + \ln(2t^2) = \ln(8)$

$\ln(4t) = \ln(8)$

says $4t = 8$

$t = 2$

b) $e^{2x+3} - 7 = 0$

$e^{2x+3} = 7$

$2x + 3 = \ln(7)$

$x = \dfrac{\ln(7) - 3}{2}$

4) ~~a)~~ 2

$f(-1) = 2$ $\quad \lim_{x\to 1^+} f(x) = 1$ $\quad \lim_{x\to -1} f(x) = 1$

$f(1) = 2.5$ $\quad \lim_{x\to 1^-} f(x) = 2.5$ $\quad \lim_{x\to 2} f(x) = \infty.$

$f(2) = \text{DNE}$ $\quad \lim_{x\to 1} f(x)$ DNE

5) $$\lim_{x\to 3} \frac{x^2-9}{x^2+2x-3} = \frac{0}{12} = 0.$$

(substitution)

$$\lim_{x\to 1} e^{x^3-x} = e^{\left(\lim_{x\to 1} x^3-x\right)} = e^0 = \underline{1}$$

$$\lim_{r\to 9} \frac{\sqrt{r}}{(r-9)^4} = \infty$$

$\sqrt{r} \to 3$, and the denominator approaches 0, and is always positive.

$$\lim_{x\to\infty} \frac{\sqrt{x^2-9}}{2x+6} = \lim_{x\to\infty} \frac{\sqrt{\frac{x^2-9}{x^2}}}{2+6/x} = \lim_{x\to\infty} \frac{\sqrt{1-\frac{9}{x^2}}}{2+6/x} = \boxed{\frac{1}{2}}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2-9}}{2x+6} = \lim_{x\to-\infty} \frac{\frac{\sqrt{x^2-9}}{-\sqrt{x^2}}}{2+6/x} = \lim_{x\to-\infty} -\frac{\sqrt{1-9/x^2}}{2+6/x} = \boxed{-\frac{1}{2}}$$

$$\lim_{x\to 3} \frac{\sqrt{x+6}-x}{x^3-3x^2} = \lim_{x\to 3} \frac{(\sqrt{x+6}-x)(\sqrt{x+6}+x)}{x^2(x-3)(\sqrt{x+6}+x)} = \lim_{x\to 3} \frac{-(x^2-x-6)}{x^2(x-3)(\sqrt{x+6}+x)}$$

$$= \lim_{x \to 3} \frac{-(x-3)(x+2)}{x^2(x-3)(\sqrt{x+6}+x)} = \lim_{x \to 3} \frac{-(x+2)}{x^2(\sqrt{x+6}+x)}$$

$$= \boxed{\frac{-5}{54}}$$

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6) $\lim_{x \to \pi/4^-} f(x) = \lim_{x \to \pi/4^-} \sin(x) = \frac{\sqrt{2}}{2}$

$\lim_{x \to \pi/4^+} f(x) = \lim_{x \to \pi/4^+} \cos(x) = \frac{\sqrt{2}}{2}$.

So $\lim_{x \to \pi/4} f(x) = \frac{\sqrt{2}}{2} = f(\pi/4)$.

So f is continuous at $\pi/4$.

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7) a) $\lim_{h \to 0} \frac{(2+h)^2 - 1 - (2^2 - 1)}{h} = f'(2)$

$$= \lim_{h \to 0} \frac{4 + 4h + h^2 - 1 - 4 + 1}{h}$$

$$= \lim_{h \to 0} \frac{h(4+h)}{h}$$

$$= \lim_{h \to 0} 4 + h = \boxed{4}$$

7b) $$f'(1) = \lim_{h\to 0} \frac{\sqrt{1+h} - \sqrt{1}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{1+h} - 1)}{h} \cdot \frac{(\sqrt{1+h} + 1)}{(\sqrt{1+h} + 1)}$$

$$= \lim_{h\to 0} \frac{1+h-1}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \boxed{\frac{1}{2}}$$

7c) $$f'(x) = \lim_{h\to 0} \frac{\frac{x+h-1}{x+h-2} - \frac{x-1}{x-2}}{h}$$

$$= \lim_{h\to 0} \frac{(x+h-1)(x-2) - (x+h-2)(x-1)}{(x+h-2)(x-2) \cdot h}$$

$$= \lim_{h\to 0} \frac{x^2 + xh - x - 2x - 2h + 2 - (x^2 + xh - 2x - x - h + 2)}{(x+h-2)(x-2) \cdot h}$$

$$= \lim_{h\to 0} \frac{-1}{(x-h-2)(x-2)} = \boxed{\frac{-1}{(x-2)^2}}$$

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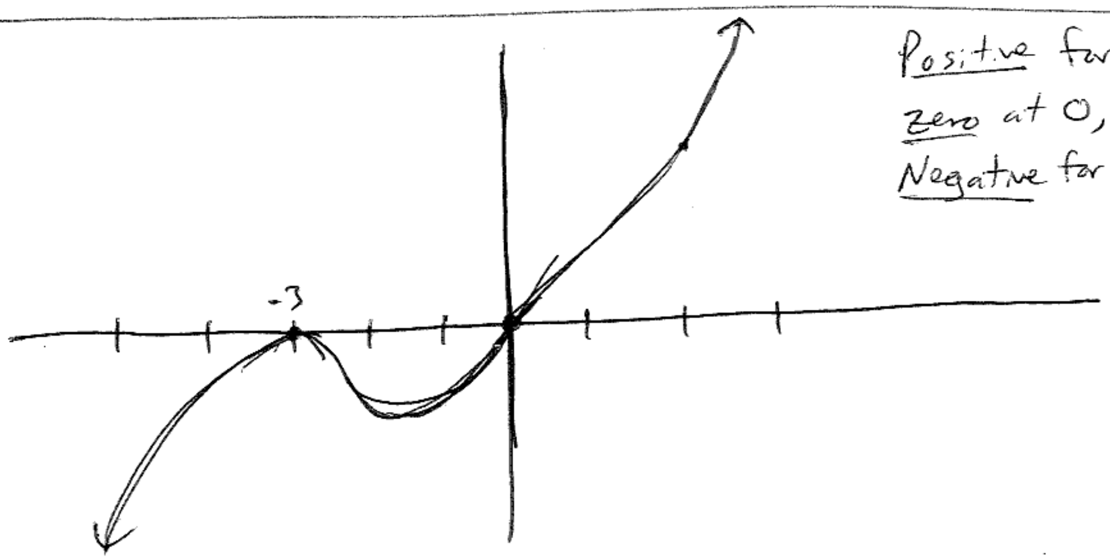
8)



Positive for $x > 0$,
Zero at $0, -3$,
Negative for $x < 0$.