

Exam 1

Answer each problem completely and show all work in the space provided to get full credit. You may use the back of the page, but make a note of it. Carefully read the directions for each problem.

Problem 1.

(1) Let $f(x) = (2x^2 + 1)\sqrt{x - 1}$ and let $g(x) = \sqrt{x - 1}$. Simplify $(f/g)(x)$, and state the domain.

(2) For $F(x) = \sqrt[3]{1 + \arcsin(x^2 + 1)}$, write the function as a composition of three functions. That is, find $f(x)$, $g(x)$, and $h(x)$ so that $F(x) = (f \circ g \circ h)(x)$.

Problem 2. Find the exact value of each expression.

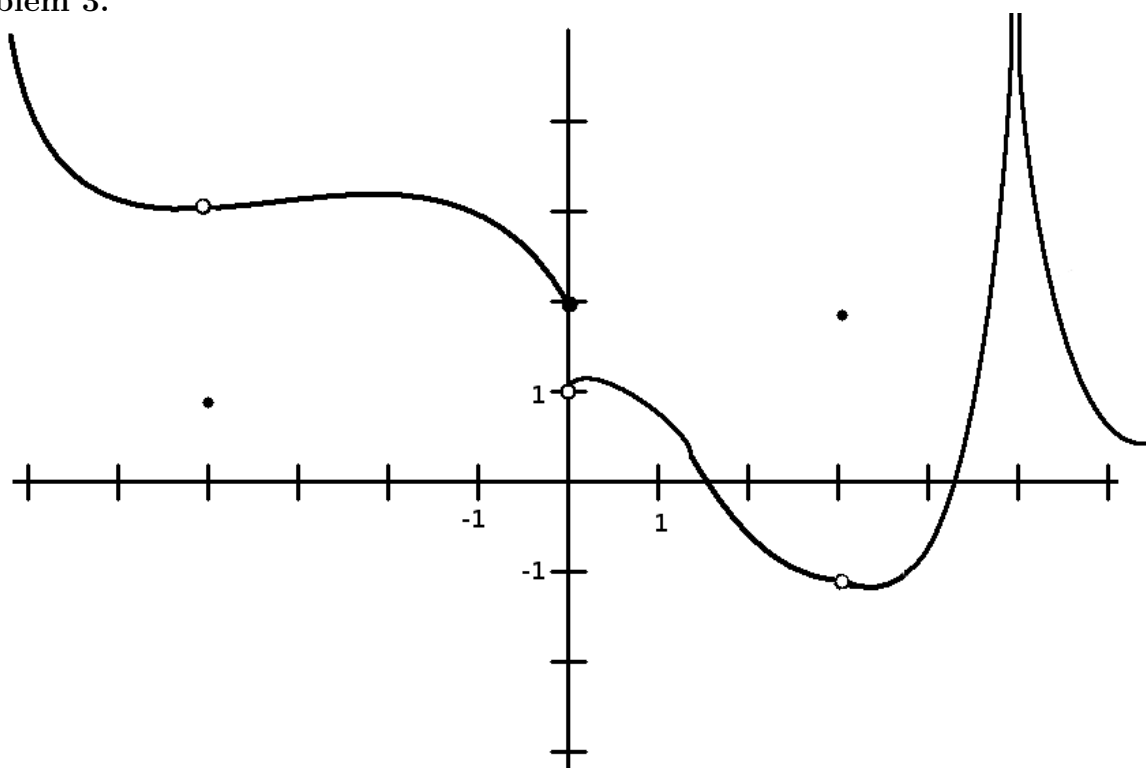
a) $\ln(\ln(e^{e^{12}})) =$ _____

b) $\sin(\arccos(-1)) =$ _____

c) $\ln(15e^2) - \ln(3) - \ln(5) =$ _____

d) $\arctan(\tan(\pi/3)) =$ _____

Problem 3.



Using the graph above, determine the following:

a) $\lim_{x \rightarrow 0^-} f(x) =$ _____

b) $\lim_{x \rightarrow 0^+} f(x) =$ _____

c) $\lim_{x \rightarrow 0} f(x) =$ _____

d) $\lim_{x \rightarrow -4} f(x) =$ _____

e) $f(-4) =$ _____

f) $\lim_{x \rightarrow 5} f(x) =$ _____

g) List and classify the discontinuities of this function. (For each one, write something like “*Jump* discontinuity at $x = 95$.”)

Problem 4. Find the following limits.

(1) $\lim_{x \rightarrow 4} 3x^2 - 2x - 1$

(2) $\lim_{x \rightarrow 25} \frac{x - 25}{\sqrt{x} - 5}$

(3) $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^{10} - 3x^5 + 1}}{2x^5 + x^4}$

(4) $\lim_{x \rightarrow -\infty} \arctan\left(\frac{2x^3 - 5x + 1}{2x^3 + 2000}\right)$

Problem 5. Find all vertical and horizontal asymptotes of

$$f(x) = \frac{2x^2 - 4x - 6}{x^2 - 9}$$

(Remember to use limits to verify the asymptotes.)

Problem 6. Find a value of k , if possible, that makes the function

$$f(x) = \begin{cases} 2x^3 + k & x \leq 2 \\ x^2 - 4x + 7 & x > 2 \end{cases}$$

continuous. (Show *why* this value works for full credit.)

Problem 7. For each function, write the limit you would evaluate to obtain the derivative. (*Don't* try to evaluate the limit!)

a) $f(x) = e^{2x-5} + x$

b) $g(x) = \sin(x^2 + 3x)$

Problem 8. Find $f'(2)$, using the limit definition of the derivative, for $f(x) = x^2 + 3x$.

Problem 9. On the blank axis provided below, sketch the graph of the derivative of the given function. (*Note:* The derivative doesn't exist at $x = 3$. So *your* graph should have hole there.)

