DISCRETE OPTIMIZATION: PROBLEM SET 6

Problem 1. Solve the following stable marriage problem. There are 5 men, {A, B, C, D, E} and five women {a, b, c, d, e}. Their preferences are as follows (each person lists those of the opposite sex in order from most to least preferred.)

A: (c, d, a, b, e)	a: (A, C, D, E, B)
B: (a, c, b, d, e)	b: (A, B, D, C, E)
C: (a, e, b, d, c)	c: (D, E, C, A, B)
D: (b, a, c, e, d)	d: (C, B, A, E, D)
E: (b, c, a, d, e)	e: (A, B, D, E, C)

Problem 2. Find and explain an example of a stable marriage problem that illustrates that the Gale-Shapley algorithm gives the proposers the advantage. That is, an example where the algorithm finds a stable matching that gives each proposer near their best choice, and each of those proposed to get near their worst choice. If possible, make the example general, in the sense that there are n men and n women, where n is unspecified.

Problem 3. The *stable roommates problem* is similar to the stable marriage problem, except instead of matching one type to another (i.e., men to women), any vertex can be matched to any other. Specifically, there are $2n$ vertices, and each has a ranking of the other $2n-1$ vertices. A perfect matching of these vertices is stable if there is no pair of vertices x, y where x prefers y to its current match and y prefers x to its current match. Unlike the stable marriage problem, there are instances of the stable roommates problem where every possible matching is unstable. Find and explain a (small) example of this problem that has no stable matching.

Problem 4. There are five jobs, v, w, x, y, z and five mechanics A, B, C, D, E. every mechanic can do every job, but each takes different amounts of time for each job. The table below shows how much time each mechanic needs for each job. You are the shop manager, and want to minimize the total amount of time spent. Solve, and explain how to solve, this weighted assignment problem.

	v	w	x	y	z
A	11	5	21	7	18
B	17	4	20	9	25
C	4	1	3	2	4
D	6	2	19	3	9
E	19	7	23	18	26