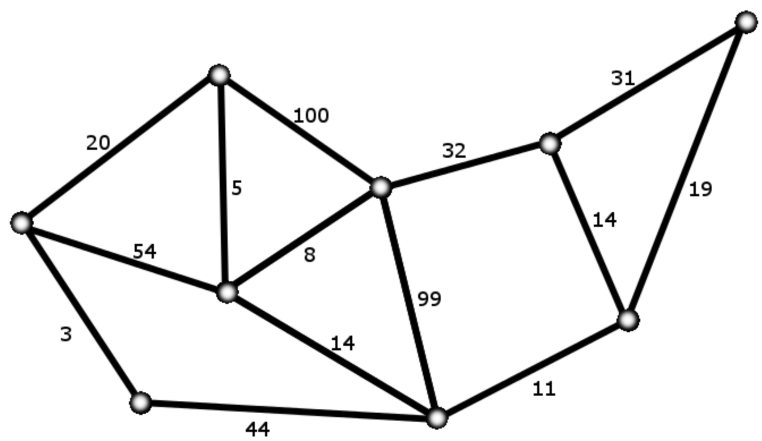
DISCRETE OPTIMIZATION: PROBLEM SET 4

Problem 1. For the graph pictured below, find the minimum weight spanning tree using

(1) Prim's algorithm starting from the marked vertex
(2) Kruskal's algorithm



Problem 2. Call the *reliability* of a graph the minimum reliability of any pair of vertices. (Recall: the reliability of a path P is the minimum weight of the edges of P, and the reliability a pair of vertices is the maximum reliability of all paths connecting the two vertices.) Find the reliability of the graph above, and point out the edge that achieves this value.

Problem 3. Prove that if G is a graph where all of the edge weights are distinct, the minimum spanning tree is unique. (Hint: Try contradiction.)

Problem 4. Suppose we have a graph G, and a minimum spanning tree T. Now we reduce the edge weight of e, one of the edges *not* in T. How can you find the minimum spanning tree in the new graph? (Don't say to run a spanning tree algorithm on the new graph. Use the information we know about T.)

Problem 5. Suppose we have a graph G, and a minimum spanning tree T. Suppose we also have some subset S of the vertices of G. Consider the graph $G[S]$ formed by deleting vertices and edges outside of S, and similarly for $T[S]$ (these are called *subgraphs induced by* S).

(1) Show that if $T[S]$ is connected, then it is a minimum spanning tree of $G[S]$.
(2) Is it true that if $G[S]$ is connected, then $T[S]$ is a minimum spanning tree of $G[S]$? (explain)