DISCRETE OPTIMIZATION: PROBLEM SET 3

Problem 1. Describe an algorithm that will find a cycle of negative weight in a directed graph if it exists, and can tell if there isn't one.

Problem 2. Find an application of the algorithm from problem 1. That is, describe a real-world situation where finding cycles of negative weight would be important or beneficial.

Problem 3. Prove that the following statement is true, and explain why it shows that the Bellman-Ford algorithm finds the length of the shortest path from s to any vertex v.

Claim: For $0 \leq k \leq n$ and each vertex v,

$$f_k(v) = \min_{\text{paths } P} \{\ell(P) \mid P \text{ is an } s - v \text{ path with at most } k \text{ edges.}\}$$

(Hint: Use induction on k, and the condition for getting the new function value from the old.)

Problem 4. Draw the graph corresponding to the following knapsack problem, and use the Bellman-Ford algorithm to solve it, showing the tree that results.

You have a (small) knapsack that can carry at most 8 pounds. Which of the items do you take to maximize value?

item	weight	value
a	5	4
b	3	7
c	2	3
d	2	5
e	1	4

Problem 5. Draw the PERT graph for the following schedule for building a house. Use the Bellman-Ford algorithm to find a critical path. How much could you reduce the time needed if you are allowed to reduce the time for exactly one task? (That is, you can change the time required for any one task, by any amount. Which one do you shorten, and by how much, to have the greatest impact.)

task	time needed	complete before
1. groundwork	1	2
2. foundation	4	3
3. walls	10	4,6,7
4. exterior plumbing	4	5,9
5. interior plumbing	5	10
6. electricity	7	10
7. roof	6	8
8. finish outer walls	7	9
9. exterior painting	9	14
10. panelling	8	11,12
11. floors	4	13
12. interior painting	5	13
13. finish interior	6	
14. finish exterior	2	