DISCRETE OPTIMIZATION: FINAL EXAM

Complete each of the following problems, and clearly write explanations for each solution. You may use WebSim and the textbook, but please try not to use any other sources. You are NOT permitted to work with other students on these problems.

Problem 1. Write an LOP that solves the following problem. You do not need to solve the problem.
A farmer owns 1000 acres of land, and has three options for using it. He can use it for conservation, for farming, or for development. It costs him \$1 and acre to register land for conservation, and he gains \$30 in tax credits for doing so. If he farms, it costs him \$50 an acre for seed, and he can make \$190 an acre by selling the vegetables. If he develops it, there is an \$85 an acre permit fee, and he can make \$290 an acre in rent. He has \$40,000 to spend, and 75 workers, each of whom can work 2,000 hours. It takes 12 hours of labor per acre of conservation, farming takes 240 hours per acre, and development takes 180 hours of work per acre. What should he do to maximize his income?

Problem 2. Solve the following LOP graphically. That is, draw the feasible region, find all of the extreme points, and determine which point has the maximum value for the objective function.

$$\begin{array}{llrcrcl}
Max. & z = & 4x_1 & + & 5x_2 & & \\
s.t. & & 2x_1 & + & x_2 & \leq & 6 \\
 & & x_1 & + & x_2 & \leq & 4 \\
 & & x_1 & + & 2x_2 & \leq & 6 \\
 & & & & & & \\
\& & & x_1 & , & x_2 & \geq & 0
\end{array}$$

Problem 3. Solve the following LOP with the simplex method. (WebSim recommended) for each simplex step, beginning with the initial tableau ($k = 0$) and ending with the final, write (or print out) the tableau $T^{(k)}$, the basis and parameters $\beta^{(k)}, \pi^{(k)}$, and basic solution and objective value $\mathbf{x}^{(k)}, z^{(k)}$. Between tableau, write the pivot operation $i \mapsto j$. Finally, read the dual solution $\mathbf{y}$ from the last tableau.

$$\begin{array}{llrcrcl}
Max. & z = & 4x_1 & + & 5x_2 & & \\
s.t. & & 2x_1 & + & x_2 & \leq & 6 \\
 & & x_1 & + & x_2 & \leq & 4 \\
 & & x_1 & + & 2x_2 & \leq & 6 \\
 & & & & & & \\
\& & & x_1 & , & x_2 & \geq & 0
\end{array}$$

Problem 4. Consider the following LOP:

$$
\begin{array}{llrcrcrcr}
Max. & z = & 3x_1 & + & 2x_2 & - & 3x_3 & & \\
s.t. & & x_1 & + & x_2 & - & 2x_3 & \leq & -1 \\
 & & & & -x_2 & - & 2x_3 & \leq & -1 \\
 & & -x_1 & + & x_2 & - & 3x_3 & \leq & -2 \\
 & & & & & & & & \\
\& & & x_1 & , & x_2 & , & x_3 & \geq & 0
\end{array}
$$

(1) Write the dual LOP to this problem.
(2) Show that $\mathbf{x} = (1, 0, 1)$ and $\mathbf{y} = (3, 1, 0)$ are not an optimal pair **using complementary slackness**.
(3) Show that $\mathbf{x} = (1, 0, 1)$ and $\mathbf{y} = (3, 0, 0)$ are not an optimal pair **using complementary slackness**.

Problem 5. Find the saddle points of the following payoff matrix. What's the value of the game, and what are optimal strategies for the row and column players?

$$
\begin{pmatrix}
4 & 2 & -3 & -1 \\
-1 & 2 & 0 & 1 \\
1 & 3 & -2 & 0 \\
-3 & 2 & 1 & 2 \\
0 & 3 & 4 & 2
\end{pmatrix}
$$

Problem 6. Solve the following game. That is, find the optimal strategies for column and row, and the value of the game. *Hint: Reduce by dominance first.*

$$
\begin{pmatrix}
3 & 2 & 1 & 0 \\
0 & 1 & 2 & 0 \\
1 & 0 & 2 & 1 \\
3 & 1 & 2 & 2
\end{pmatrix}
$$

Problem 7. The problem k-COLOR takes a (let's assume connected) graph, and returns "yes" if there is a proper coloring with k colors, and no otherwise. We know from class that 3-COLOR is NP-complete. Show that 2-COLOR can be decided in polynomial time. That is, give me an algorithm that tries to properly color the graph, and the algorithm only fails if there is no 2-coloring. (You probably want to use the fact that a graph is 2-colorable if and only if there are no cycles of odd length...)