SOLUTION OF HOMEWORK PROBLEM 4 FROM PAGE 18

Problem 4, page 18. John is concerned by a long-term weather report predicting a hotter than normal summer. He will be working at a forest fire lookout in a remote site and wants to be sure of having an adequate beer supply. He can haul 12 cases to the site in June at a cost of \$8 per case. A friend (Bud) will visit him in July and can bring up to 20 cases at \$9.50 per case. Another friend (Busch) with a smaller car can bring 8 cases in July and another 8 cases in August, at \$9 per case. John estimates he will *need* 10 cases in June, 12 cases in July, and 15 cases in August to cope with the increasing heat. How should he schedule the deliveries to keep up with his thirst at the least cost?

Solution: Since there are four supply trips, each of limited capacity, we must posit 4 supply sources:

$$\begin{aligned} S_1 &= \text{John in June,} \\ S_2 &= \text{Bud in July,} \\ S_3 &= \text{Busch in July,} \\ S_4 &= \text{Busch in August.} \end{aligned}$$

There are obviously three demand periods:

$$\begin{aligned} D_1 &= \text{June,} \\ D_2 &= \text{July,} \\ D_3 &= \text{August,} \end{aligned}$$

If $1 \leq i \leq 4$ and $1 \leq j \leq 3$ then define

$$x_{ij} = \text{number of cases from source } S_i \text{ to be consumed during demand period } D_j.$$

The total cost of all this beer is

$$\begin{aligned} z = 8x_{11} &+ 8x_{12} + 8x_{13} \\ &\infty x_{21} + 9.5x_{22} + 9.5x_{23} \\ &\infty x_{31} + 9x_{32} + 9x_{33} \\ &\infty x_{41} + \infty x_{42} + 9x_{43}. \end{aligned}$$

The costs of ∞ reflect the impossibility of John consuming beer before it is delivered to him. If we minimize z then obviously we must have $x_{21} = 0, x_{31} = 0, x_{41} = 0,$ and $x_{42} = 0$. The supply and demand constraints then assume their standard forms:

$$
\begin{aligned}
x_{11} + x_{12} + x_{13} &\le 12, \\
x_{21} + x_{22} + x_{23} &\le 20, \\
x_{31} + x_{32} + x_{33} &\le 8, \\
x_{41} + x_{42} + x_{43} &\le 8, \\
x_{11} + x_{21} + x_{31} + x_{41} &= 10, \\
x_{12} + x_{22} + x_{32} + x_{42} &= 12, \\
x_{13} + x_{23} + x_{33} + x_{43} &= 15,
\end{aligned}
$$

These should be understood in light of the vanishing of the variables we have already mentioned. So the associated linear programming problem is to minimize

$$
\begin{aligned}
z = 8x_{11} &+ 8x_{12} + 8x_{13} \\
&+ 9.5x_{22} + 9.5x_{23} \\
&+ 9x_{32} + 9x_{33} \\
&\qquad + 9x_{43}.
\end{aligned}
$$

subject to the constraints

$$
\begin{aligned}
x_{11} + x_{12} + x_{13} &\le 12, \\
x_{22} + x_{23} &\le 20, \\
x_{32} + x_{33} &\le 8, \\
x_{43} &\le 8, \\
x_{11} \qquad\qquad\qquad &= 10, \\
x_{12} + x_{22} + x_{32} \qquad &= 12, \\
x_{13} + x_{23} + x_{33} + x_{43} &= 15, \\
x_{ij} &\ge 0, 1 \le i \le 4, 1 \le j \le 3.
\end{aligned}
$$