Solutions to HW 5.5 – 5.10

5.5 Let P be the price of a put option (ie. option to sell a security whose present price is S) for the amount K. Which of the following are necessarily true? a) $P \le S$ b) $P \le K$.

Answer By 5.2 $C \le S$ where C is the price of a call option ie. $0 \le S - C$. Adding P to both sides yields

$P \le S + P - C = Ke^{-rt}$ by the put-call option parity formula

$\le K$ since $e^{-rt} \le 1$ $(r \ge 0, t \ge 0)$

$\therefore$ we must have $P \le K$ ie. b). a) doesn't seem necessary.

5.6 Let P be the price of a (K,t) put option (to sell a security whose present price is S). Show that $P \ge Ke^{-rt} - S$

Answer By the put-call option parity formula

$P = Ke^{-rt} - S + C$. But $C < 0$ leads to an arbitrage since if one buys a negatively priced contract which does not obligate you in any way one gains money. Thus $C \ge 0$. So $Ke^{-rt} - S + C \ge Ke^{-rt} - S$. Hence $P \ge Ke^{-rt} - S$. ✓

5.7 Suppose $S + P - C > Ke^{-rt}$. Show that selling one share of stock, selling one put option to A and buying one call option from B is an arbitrage.

Answer. The gain at time 0 of this transaction is $S + P - C$. This is positive under the supposition, so deposit it in the bank. At time t one has $(S+P-C)e^{rt} > K$ in the bank. Party A may wish to exercise the put option (if $S(t) < K$) and make you buy a share from party A for the above market price K. If so, withdraw K from the bank and buy the share from A. One has $(S+P-C)e^{rt} - K > 0$ still in the bank and one can return the share that one sold short. If $S(t) \ge K$ then party A will not exercise the put option. But you must return the share you sold short to the brokerage. Thus you should exercise your call option and buy a share of stock from party B at the price $K \le S(t)$. Use K from the bank, leaving $(S+P-C)e^{rt}-K>0$ still there. Return the share you bought to the brokerage.

5.8 Use the LOP to prove the Put-Call option parity formula.

Answer investment #1: buy 1 share of stock, sell one (K,t) call option to A
investment #2: deposit Ke^{-rt} in bank, sell one (K,t) put option to B

$C_1 = S - C \qquad C_2 = Ke^{-rt} - P$

Payoff at time t for investment #1

if $S(t) > K$ then party A will want to exercise the call option & buy 1 share of stock for the below market price K. Give A one share (bought at time 0) and receive K.

if $S(t) \le K$ then keep one share of stock

Payoff at time t for investment #2

if $S(t) \ge K$ then withdraw K from bank

if $S(t) \le K$ then party B will want to exercise the put option & sell 1 share of stock to you for the above market price of K. Fortunately you have exactly K in the bank; withdraw it from the bank and pay it to party B & receive from him one share of stock.

Since whatever happens to the stock price $S(t)$ the payoff #1 = payoff #2 by LOP either $C_1 = C_2$ or there is an arbitrage. Since there should not be an arbitrage we have $C_1 = C_2$ i.e. $S - C = Ke^{-rt} - P$.

5.9 A European call and put option on the same security both expire in 3 months, both have a strike price of 20, and both sell for the price 3. If the nominal continuously compounded interest rate is 10% and the stock price is currently 25, identify an arbitrage.

Answer $t = .25$ (years). $r = .1$, $K=20$, $C=P=3$, $S=25$

$Ke^{-rt} = 20e^{-.1(.25)} = 19.51$ $S+P-C = 25 > 19.51$

arbitrage instructions: sell (short) one share of stock (time 0) for 25. Sell one put option (for 3) and buy one call option (for 3). At time 0 then the net gain is 25. At time $t=.25$ let $S(.25)$ be the price of the stock. There are two alternatives.

If $S(.25) \geq 20$ then exercise the call option and buy a share of stock for 20, return the share to the brokerage. The put option will not be exercised. You clear at least 5.

If $S(.25) < 20$ then the put option will be exercised (by the person who bought it) and you will be forced to buy a share of stock from him for 20. Again, return the share to the brokerage. Again you clear at least 5.

5.10 Let C_a and P_a be the costs of American call and put options (respectively) on the same security, both having the same strike price K and expiration time t. If S is the present price of the security, relate C_a, P_a, S, K, and e^{-rt}.

Answer By Prop. 5.2.1 $C_a = C$ = the price of a (K,t) European call option. The text (p.68) claims $P_a \geq P$ = the price of a (K,t) European style put option. Hence $P_a \geq P = C - S + Ke^{-rt}$ $= C_a - S + Ke^{-rt}$. Thus $P_a \geq C_a - S + Ke^{-rt}$.