Solutions to HW Problems (continued)

<u>5.25</u> <u>b</u> $P(1<Y_1<2.5) = \int_1^{2.5} f_1(y_1)\,dy_1 = \int_1^{2.5} e^{-y_1/2}\,dy_1$

$= -2e^{-y_1/2}\Big|_1^{2.5} = -2[e^{-1.25}-e^{-0.5}]$

$= 2[e^{-0.5}-e^{-1.25}] = 0.64$

$P(1<Y_2<2.5) = 0.64$ since $f_2(y)=f_1(y)$.

<u>c</u> For what values of y_2 is the conditional density $f(y_1|y_2)$ defined?

$$f(y_1|y_2) = \frac{f(y_1,y_2)}{f_2(y_2)} \quad \text{provided } f_2(y_2)>0$$

i.e. $y_2>0$.

<u>d</u> For any $y_2>0$ what is the conditional density function for Y_1 given that $Y_2=y_2$?

$$f(y_1|y_2) = \frac{1}{e^{-y_2/2}}\begin{cases} e^{-(y_1+y_2)/2} & \text{if } y_1>0 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} e^{-y_1/2} & \text{if } y_1>0 \\ 0 & \text{otherwise} \end{cases}$$

<u>e</u> For any $y_1>0$ what is the conditional density function for Y_2 given that $Y_1=y_1$?

$$f(y_2|y_1) = \frac{f(y_1,y_2)}{f_1(y_1)} = \frac{1}{e^{-y_1/2}}\begin{cases} e^{-(y_1+y_2)/2} & y_2>0 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} e^{-y_2/2} & y_2>0 \\ 0 & \text{otherwise} \end{cases}$$

<u>f</u> For $y_2>0$ compare $f(y_1|y_2)$ to $f_1(y_1)$.

They are equal for all y_1

g What does this imply about the marginal probability that $a<Y_1\le b$ and the conditional probability that $a<Y_1\le b$ given $Y_2=y_2$ (a fixed # >0)?

Answer: In this case these two probabilities are equal for any $a<b$.

5.100 Let $Y_1 = Z$ have a standard normal distribution and $Y_2 = Z^2$

a $E(Y_1) = E[Z] = 0$ (since $\mu = 0$ for a standard normal)

$E(Y_2) = E(Z^2) = E(Z^2) - E(Z)^2 = V(Z) = 1$ (since $\sigma^2 = 1$ for a standard normal).

b $E(Y_1 Y_2) = E(Z \cdot Z^2) = E(Z^3) = \int_{-\infty}^{\infty} z^3 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$

$= \int_{-\infty}^{\infty} g(z)\,dz$ where $g(-z) = -g(z)$ for all z

$= 0$ (integral of an odd function over $-\infty$ to ∞)

c $Cov(Y_1, Y_2) = E(Y_1 Y_2) - E(Y_1)E(Y_2) = 0 - 0 \cdot 1 = 0.$

d $P(Y_2 > 1 \mid Y_1 > 1) = 1$ since $Y_2 = Y_1^2$

But $P(Y_2 > 1) = P(Y_1^2 > 1) = 1 - P(-1 \le Y_1 \le 1)$

$= 1 - .6826 = .3174$

Since $P(Y_2 > 1 \mid Y_1 > 1) \neq P(Y_2 > 1)$

Y_1, Y_2 are not independent.