Solutions to Homework Problems

2.102 E_1 = person contracts disease I in his/her lifetime

E_2 = " " " II " " "

$P(E_1) = .1$, $P(E_2) = .15$, $P(E_1 \cap E_2) = .03$

a. $E_1 \cup E_2$ = person contracts disease I or disease II or both

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = .1 + .15 - .03 = .22$

b. $P(E_1 \cap E_2 \mid E_1 \cup E_2) = \dfrac{P(E_1 \cap E_2 \cap (E_1 \cup E_2))}{P(E_1 \cup E_2)} = \dfrac{P(E_1 \cap E_2)}{P(E_1 \cup E_2)} = \dfrac{.03}{.22}$

$= .1\overline{36}$

2.104 Prove that $P(A \cap B) \geq 1 - P(\bar{A}) - P(\bar{B})$

to see $P(A) + P(B) - P(A \cap B) = P(A \cup B) \leq 1$

$P(A) + P(B) - 1 \leq P(A \cap B)$ since $P(\bar{A}) = 1 - P(A)$

$1 - P(\bar{A}) - P(\bar{B}) \leq P(A \cap B)$ //

2.125 A = diagnostic test is positive (i.e. patient seems to have disease)

B_1 = patient has disease

$B_2 = \bar{B}_1$ = patient does not have disease

$P(A \mid B_1) = .9$, $P(\bar{A} \mid B_2) = .9$ $P(B_1) = .01$

$$P(B_1 \mid A) = \frac{P(B_1 \cap A)}{P(A)} = \frac{P(A \mid B_1)\,P(B_1)}{P(A \mid B_1)\,P(B_1) + P(A \mid B_2)\,P(B_2)}$$

$$= \frac{(.9)(.01)}{(.9)(.01) + (.1)(.99)} = \frac{0.009}{0.009 + 0.099} = \frac{0.009}{0.108} = .0\overline{83}$$

Am I surprised?

Is this diagnostic test reliable? No.

2.129 A = subject reacts negatively $\bar{A}$ = subject reacts positively

B_1 = subject is male $B_2 = \bar{B}_1$ = subject is female.

$P(\bar{A}|B_2) = .7$, $P(\bar{A}|B_1) = .4$ $P(B_1) = .25$ $P(B_2) = .75$

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2)} = \frac{(.6)(.25)}{(.6)(.25)+(.3)(.75)}$$

$$= \frac{0.15}{0.15+0.225} = \frac{0.15}{0.375} = 0.4$$

3.6 $S = \binom{5}{2}$. Each outcome in S is equally likely.

a. $X: S \to R = \{i,j\} \mapsto \max\{i,j\}$. Find $p(x)$

Range$(X) = \{2,3,4,5\}$

S	x	$p(x)$
$\{1,2\}$	2	$\frac{1}{10}$
$\{1,3\}, \{2,3\}$	3	$\frac{2}{10}$
$\{1,4\}, \{2,4\}, \{3,4\}$	4	$\frac{3}{10}$
$\{1,5\}, \{2,5\}, \{3,5\}, \{4,5\}$	5	$\frac{4}{10}$

b $Y: S \to R: \{i,j\} \mapsto i+j$ Find $p(y)$

Range$(Y) = \{3,4,5,6,7,8,9\}$

S	y	$p(y)$
$\{1,2\}$	3	$\frac{1}{10}$
$\{1,3\}$	4	$\frac{1}{10}$
$\{1,4\}, \{2,3\}$	5	$\frac{2}{10}$
$\{1,5\}, \{2,4\}$	6	$\frac{2}{10}$
$\{2,5\}, \{3,4\}$	7	$\frac{2}{10}$
$\{3,5\}$	8	$\frac{1}{10}$
$\{4,5\}$	9	$\frac{1}{10}$

3.9 0 = accounting entry is correct (95%) 1 = entry is an error (5%)

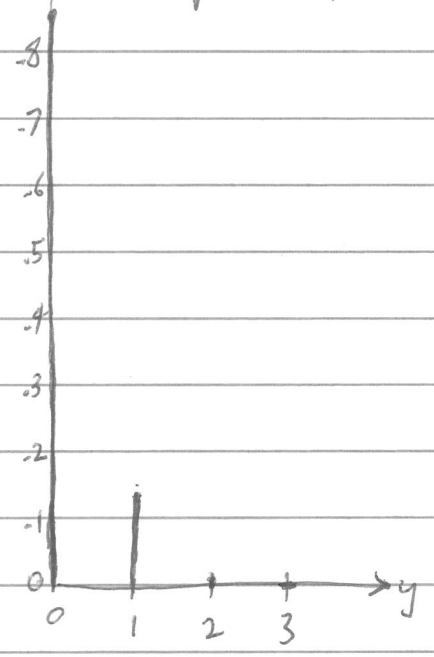
$S = \{0,1\}^3$ sample space for outcomes of 3 entries checked.

a. $Y: S \to R: (i,j,k) \mapsto i+j+k$ = # of errors

Find $p(y)$. Range$(Y) = \{0,1,2,3\}$.

S	y	p(y)
(0,0,0)	0	$(.95)^3 = .857375$
(1,0,0), (0,1,0), (0,0,1)	1	$3(.95)^2(.05) = .135375$
(1,1,0), (1,0,1), (0,1,1)	2	$3(.95)(.05)^2 = .007125$
(1,1,1)	3	$(.05)^3 = .000125$

b. Probability histogram for p(y)



c. $P(Y > 1) = p(2) + p(3) = .007125 + .000125 = 0.00725$