Solutions to Homework Problems

2.55

a. N = set of 90 nurses employed by the hospital

$\binom{N}{10}$ = set of all 10 member subsets of N, ie all selections of 10 nurses to conduct a survey

b. M = set of 20 male nurses. $M \subset N$

$F = N \setminus M$ = set of 70 female nurses

$E = \{ A \in \binom{N}{10} \mid |A \cap M| = 4 \}$ = set of selections having exactly 4 male nurses.

Define $f: \binom{M}{4} \times \binom{F}{6} \longrightarrow E : (B, C) \longmapsto B \cup C$. f is 1-1 & onto.

So $\left|\binom{M}{4} \times \binom{F}{6}\right| = |E| = \binom{20}{4}\binom{70}{6}$

$$P(E) = \frac{|E|}{\left|\binom{N}{10}\right|} = \frac{\binom{20}{4}\binom{70}{6}}{\binom{90}{10}} = \frac{20!}{4!\,16!} \; \frac{70!}{6!\,64!} \; \frac{10!\,80!}{90!}$$

$$= \frac{20 \cdot 19 \cdot \cancel{18}^{2} \cdot \cancel{17}}{1 \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{4}} \cdot \frac{70 \cdot 69 \cdot 68 \cdot 67 \cdot 66 \cdot 65}{\cancel{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}} \; \frac{\cancel{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \cdot \cancel{7} \cdot \cancel{8} \cdot \cancel{9}^{3} \cdot \cancel{10}}{\cancel{90}_{9} \cdot 89 \cdot 88 \cdot 87 \cdot 86 \cdot \cancel{85}_{5} \cdot \cancel{84}_{12} \cdot 83 \cdot 82 \cdot 81}$$

$$= \frac{1{,}132{,}365{,}325}{10{,}197{,}229{,}023} \doteq 0.111046$$

2.64

$\hat{6} = \{1, 2, 3, 4, 5, 6\}$

S = set of all possible outcomes of six rolls of a die $= \hat{6}^{\hat{6}}$

If $f \in S$ and $1 \le i \le 6$ then $f(i)$ = number obtained on ith roll

$E = \{ f \in S \mid \text{range}(f) = \hat{6} \} = \text{surj}(\hat{6}, \hat{6}) = \text{bij}(\hat{6}, \hat{6})$

$$P(E) = \frac{|E|}{|S|} = \frac{6!}{6^6} = \frac{\cancel{6} \cdot 5 \cdot \cancel{4}^{2} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{\cancel{6} \cdot \cancel{6} \cdot 6 \cdot 6 \cdot \cancel{6}_{3} \cdot \cancel{6}_{3}} = \frac{5}{36 \cdot 9} = \frac{5}{324} \doteq 0.015432$$

2.76

S = set of all plumbing jobs done in a community

U = set of all plumbing jobs which were deemed unsatisfactory and resulted in a complaint

A= set of of plumbing jobs done by plumber A.

$P(U) = .1 \quad P(A) = .4$

"half the complaints dealt with plumber A" means

$P(A|U) = .5$

a. $P(U|A) = \dfrac{P(U \cap A)}{P(A)} = \dfrac{P(A|U)P(U)}{P(A)} = \dfrac{(.5)(.1)}{.4} = 0.125$

b. $P(\bar{U}|A) = \dfrac{P(\bar{U} \cap A)}{P(A)}$ $\quad A = U \cap A \cup \bar{U} \cap A$ disjoint union

So $P(A) = P(U \cap A) + P(\bar{U} \cap A)$

$.4 = (.5)(.1) + P(\bar{U} \cap A) \quad P(\bar{U} \cap A) = .4 - .05 = .35$

$P(\bar{U}|A) = \dfrac{.35}{.4} = .875$

2.77 $\quad P(A \cap B) = .1 \quad P(A \cap \bar{B}) = .3$

$P(\bar{A} \cap B) = .27 \quad P(\bar{A} \cap \bar{B}) = .33$

a. $P(A) = P(A \cap B) + P(A \cap \bar{B}) = .1 + .3 = .4$

b. $P(B) = P(A \cap B) + P(\bar{A} \cap B) = .1 + .27 = .37$

c. $P(A \cap B) = .1$

d. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .4 + .37 - .1 = .67$

e. $P(\bar{A}) = 1 - P(A) = 1 - .4 = .6$

f. $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - .67 = .33$

g. $P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - .1 = .9$

h. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.1}{.37} = .27027$

i. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.1}{.4} = .25$

2.94 P = smoke is present

$P(A|P) = .95 \quad P(B|P) = .9 \quad P(A\cap B|P) = .88$

a. $P(A\cup B|P) = P(A|P) + P(B|P) - P(A\cap B|P)$

$= .95 + .9 - .88 = .97$

b. smoke is undetected (but present) $P(\overline{A\cup B}|P)$

$= 1 - P(A\cup B|P) = 1 - .97 = .03$

2.96 $P(A) = .5$, $P(B) = .2$, A & B independent

a. $P(A\cap B) = P(A)P(B) = (.5)(.2) = .1$

b. $P(\bar{A}\cup\bar{B}) = P(\overline{A\cap B}) = 1 - P(A\cap B) = .9$

c. $P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B}) = 1 - P(A\cup B)$

$= 1 - [P(A) + P(B) - P(A\cap B)] = 1 - [.5 + .2 - .1] = .4$

Note: $P(\bar{A}\cap\bar{B}) = P(\bar{A})P(\bar{B}) = (.5)(.8)$

so $\bar{A}$ & $\bar{B}$ are also independent