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**Title**: Banach spaces with nearly compact approximate midpoint sets

**Abstract**: Let $\alpha$ denote the Kuratowski measure of noncompactness. Given a functional $f$ (on a Banach space $X$) of unit norm and $0 < \varepsilon < 1$, the slice $S(f, 1-\varepsilon)$ consists of all $x$ in the unit ball with $f(x) > 1-\varepsilon$. Rolewicz (1987) proved that $\alpha(S(f, 1-\varepsilon)) \to 0$ as $\varepsilon \to 0$ uniformly over $f$ if and only if $X$ is reflexive and 'asymptotically uniformly convex' (AUC). We consider the analogous question for the approximate midpoint sets, $\text{lens}(x, 1+\varepsilon)$, where $x$ is a unit vector, consisting of all $y \in X$ such that $\|x \pm y\| \le 1+\varepsilon$. We prove that $\alpha(\text{lens}(x, 1+\varepsilon)) \to 0$ as $\varepsilon \to 0$ uniformly over $x$ if $X$ (not necessarily reflexive) is AUC, although the converse is false, and find an asymptotic characterization of this property.